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par
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Constraining some Neutrino Violation Properties with respect to Lepton Flavor Violation

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THESIS

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To my parents

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Abstract

Considered as one of the most promising (indirect) signals of new physics (NP) at colliders, hints of lepton flavor universality violation (LFUV) in rare B decays, both in the charged- and in the neutral-current processes, have been extensively investigated within a model-dependent approach throughout the years. Whether mediated by the exchange of (real) heavy particles (new massive bosonic mediators around the TeV scale) or hypothetical ones (leptoquarks), the deviations from the SM predictions that have been reported in both ratios $R_{K^{(*)}}$ and $R_{D^{(*)}}$ by LHCb, Belle and BaBar experiments, point towards a different behavior of the lepton flavors when it comes to their couplings with the mediators of the transition. In fact, being exclusively observed in semi-leptonic B-meson decays, NP is speculated to be mainly coupled to the third generation of quarks and leptons. In this regards, we investigate these anomalies in the framework of a model based on the extended gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ with $\beta = 1/\sqrt{3}$ whose leptonic sector should consist of no less than five lepton triplets in order to generate LFUV couplings. We work out how this set could accommodate the NP scenarios favored by global analyses performed within a model-independent approach. We show that, not only the adopted model accommodates significant NP contribution along the direction $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$, currently favored by the global fits, but also, lepton flavor violating transition might arise, provided that the NP contribution to the neutral transition $b \longrightarrow sl^+l^-$ is dominated by the exchange of both the model's heavy (exotic) neutral gauge boson Z'_μ and the (light) SM's Z_μ. For the charged current (CC) anomaly, on the other hand, the model proves able to accommodate the dominance of the vector/axial exchange, favored by the global fits, provided that the transition $b \longrightarrow cl\nu$ is mediated by the SM's gauge boson W_μ rather than the model's heavy one as its coupling with the fermions is suppressed at the desired order of energy. More precisely, the leading order contribution would stem from the matrix element that mixes an SM lepton with a massive neutrino without which, such contributing term would not appear.

Keywords: New Physics (NP), Effective Field Theories (EFTs), Standard Model Extension, Lepton Flavor Universality Violation (LFUV), Weak B- decays.

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Introduction

Being one of the most successful achievements in modern physics, the standard model (SM) provides a very elegant theoretical framework that describes all known experimental facts in particle physics (see, e.g. Refs. [1, 2]). Notwithstanding its ability to predict all sorts of reactions among particles and particle properties once its free parameters are measured, the SM is considered as an incomplete theory; in a sense that there has to be some dynamics beyond that should complement (or entirely replace) that of the SM at some energy range. In fact, despite its enormous phenomenological success, several ingredients hint towards the possibility of the existence of a new physics (NP) that could be responsible for some of the experimentally observed events, to which the SM could not provide an explanation. Effects of alternative scenarios that are built on the SM are studied only to the extent that precise measurements are performed within the SM and confronted with comparably precise experimental ones. They are referred to, collectively, as beyond the SM (BSM) theories. The SM's inability to account for the lightness of neutrinos, for instance— which does not require a deep reappraisal of the model, nevertheless— calls for a new high energy scale: the seesaw scale at around $10^9 - 10^{13}$ GeV, at which new degrees of freedom might exist [3–5]. Moreover, the lack of an explanation of gravity in a yet-to-be discovered quatized form—as the theory breaks beyond the Planck scale $\Lambda_{\text{Planck}} = G^{-1/2} \approx 10^{19}$ GeV—enforces a (natural) cut-off to the scale of validity of the SM at which it ceases to be renormalizable [6]. Dark matter (DM) constitutes also one of the issues that make the validity of the SM grind to a halt. As a matter of fact, current cosmological models agree on the fact that the DM could be made of a set of not-too heavy (nor too light) particles interacting very weakly with the normal matter, and yet, none of which is one of the SM's constituents [7]. Dark energy (DE), on its part, is way more mysterious as it is not even clear whether or not an appropriate standard particle interpretation could be adopted [8]. The problem of *Baryogenesis* is also one of the shortcomings of the SM. In fact, the dominance of matter over antimatter cannot be accounted for in the SM as this latter does not have enough $\mathcal{CP}$ violation, nor baryon number violation $\mathcal{B}$ to create an unbalanced universe. As a consequence, either new $\mathcal{CP}$ and $\mathcal{B}$-violation dynamics are required, or new degrees of freedom have to be introduced [9, 10].

Several other hints for NP that beg for a deep theoretical interpretation are to be found in its free parameters. The three SM independent gauge group couplings, for instance, which evolve with the energy, appear to meet at around $10^{14} - 10^{16}$ GeV [11, 12]. This could be interpreted by a larger gauge group—$SU(5)$ as the simplest candidate—that breaks down spontaneously to the SM's so that its unique coupling constant branches into the three couplings down from the grand unified theory (GUT) breaking scale. Moreover, the $U(1)_{\mathcal{B}+\mathcal{L}}$ violation that appears in the SM through tiny non-perturbative effects [13] and the seesaw mechanism—which is responsible for the generation of tiny majoranna masses for the left-handed neutrinos in probably the most natural way possible [3, 4]—also breaks the $\mathcal{L}$ by two units [14], even though gauge invariant couplings that violate lepton $\mathcal{L}$ or baryon $\mathcal{B}$ numbers happen to be forbidden in the SM. So, there is really no reason to expect for those accidental symmetries to be respected in nature. Another issue with the SM's free parameters is the so-called *the hierarchy puzzle* which originates in the Higgs mass as quantum corrections tend to make it heavier than it needs to be [15]. A tempting solution to this problem would be the assumption that NP arises at a scale not too far from the TeV as the radiative corrections tend to mix the two scalar sectors together (both SM's and NP's). Nevertheless, most of the mystery concerning the SM's free parameters lies in the fermionic sector as it constitutes their main source. For instance, the question of why fermions are replicated in three (nearly identical) copies is still one of the most mysterious features observed in nature especially as the number of families is not dictated by any dynamical or symmetry principle in the SM (and even beyond). Furthermore, the Yukawa couplings are arbitrary three-by-three matrices, thus the regular patterns exhibited by fermion mass generation and mixings is puzzling as there is no explanation to why it is what it is. With that being said, it is natural to expect that some physics beyond the SM could be responsible for generating all these flavor structures.

In view of all of the above, clearly the concept of new complementary dynamics that should kick in at a scale not too far from the TeV is mandatory for the theory to be remedied, which from a pragmatic perspective, would be welcomed as it would be directly accessible by the LHC. As neither the main characteristics of this NP nor its fundamental nature are known, signs of its existence are looked for either directly or inferintially from our current theoretical understanding. At the experimental plan, the investigation of the TeV scale is carried out at the high energy frontier by considering two complementary strategies performed at the LHC at CERN. The first, so-called *relativistic path*, aims at producing and detecting new heavy degrees of freedom, and thus, probing directly the scale of NP through specific signatures [16]. No signs of NP have arisen yet, however, and only SM particles have been observed so far. The second effort, *quantum path*, aims at investigating virtual effects from NP particles mediating lower energy processes and thus affecting the low-energy observables. The second category encompasses the investigation of one of the most interesting phenomena reported by particle physics experiments hinting to lepton flavor universality violation (LFUV) in semi-leptonic B decays. In fact, disagreement with the SM expectations have been revealed in specifically four anomalies appearing in ratios assessing lepton flavor universality (LFU), namely $R_{D^{(*)}}^{\tau/l}$, $(l = \mu, e)$ and $R_{K^{(*)}}^{\mu/e}$ in the flavor-changing charged current (FCCC) decays $B \longrightarrow D^{(*)} l \bar{\nu}_l$ and in the flavor-changing neutral current (FCNC) decays $B \longrightarrow K^{(*)} l^+ l^-$, respectively, for which attempts to provide a combined/coherent explanation emerged, triggering a speculation of a possible NP interpretation [17–21]. The evidence collected so far translates into deviations from τ/e (and τ/μ) universality in $b \longrightarrow c l \bar{\nu}_l$ and from μ/e universality in $b \longrightarrow s l \bar{l}$. In order to understand the pattern of these deviations in terms of NP contributions, many model-independent analyses have been performed within an effective field theory (EFT) approach corresponding to the SM at the b-quark mass scale supplemented with additional NP operators [22–26]. The global analyses were all able to interpret the deviations in terms of a shift in the short-distance Wilson coefficients that couple to the non-SM operators describing left-handed effective interactions. Even though these (model-independent) analyses are able to provide an explanation for the pattern of the anomalies in terms of NP contributions that is felt at low energies, the need for a dynamical explanation of the deviations requires the adoption of some BSM theories. Two sets of models have been proposed to account for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ simultaneously. The first constitutes models that are assumed to reproduce LFUV processes mediated by leptoquark (LQ) particles (see, e.g. Refs. [27–29]), while within models of the second set, LFUV processes are assumed to be mediated with heavy exotic gauge bosons whose couplings with the fermions depend on the generation (see, e.g. Refs. [17, 30–32]). These models feature heavy gauge bosons (W', Z') (commonly referred to as Z' models) which are supposed to mediate the transitions. The second set is based on the fact that one of the puzzling aspects of the observed anomalies is that they appear exclusively in semi-leptonic B-decays. As a matter of fact, no evidence of a deviation from the SM had been observed in semi-leptonic K or π decays, nor the purely leptonic τ decays. As a consequence, the most natural assumption to address this apparent paradox is to assume that NP is coupled mainly to the third generation of fermions. A possible choice of models to go for are the so-called 331 models which fall into the second set. They are based on the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ where the SM is embedded. Within these models, the fermions are grouped into generations where one behaves differently than the others when it comes to their couplings with the model's gauge bososns (one generation has different gauge charge assignements). This work is dedicated to the investigation of the ability of a specific version of these models to accommodate the observed B- anomalies for both the charged and the neutral flavor changing transitions in a coherent/combined way.

In doing so, a detailed description of the standard model is provided in Chapter 1, where the main focus is given to its flavor structure as the flavor theory is the core of this work. We follow with a theoretical description of the semi-leptonic rare B-decay in Chapter 2 where the main theoretical tools used for the treatment within effective field theories are presented. In Chapter 3, we present a specific BSM scenario that we adopt which is based on the extended gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (called for short 331 model). A detailed examination is provided for the model in question where LFUV arising in the gauge couplings are pointed out. The results obtained are finally compared with the global analyses performed within an EFT approach.

Chapter 1

The Standard Model

The Standard Model (SM) is a gauge theory based on the group $\mathcal{G}_{\rm SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This model provides a unified theoretical framework that describes the strong interactions of (colored) quarks and gluons, factored by $SU(3)_C$, and the weak and electromagnetic interactions that are factored by the famous Glashow-Weinberg-Salam group $SU(2)_L \otimes U(1)_Y$. These interactions occur via the exchange of the gauge group's corresponding spin-1 fields: eight massless gluons which mediate the strong interaction, one massless photon for the electromagnetic interaction and three massive bosons $W^{\pm}_{\mu}$, Z_{μ} that mediate the weak interaction. The matter content of the SM consists of fifteen fermion fields (and their anti-particles) that, based on the way they transform under the model's gauge group, are organized into five fields that have the same quantum numbers, appearing each in three different replica of *flavors* ($i = 1..3$), denoted by $\psi(A, B)_{(Y/2)}$[1]

$$\begin{aligned} &Q_i^L \sim (3,2)_{\frac{1}{6}}, \qquad u_i^R \sim (3,1)_{\frac{2}{3}}, \qquad d_i^R \sim (3,1)_{-\frac{1}{3}}, \\ &L_i^L \sim (1,2)_{-\frac{1}{2}}, \qquad l_i^R \sim (1,1)_{-1}, \end{aligned} \tag{1.1}$$

[1] A and B denote the representation under the $SU(3)_C$ and $SU(2)_L$ groups respectively, while Y is the $U(1)_Y$ charge.

where the left-handed fields are the $SU(2)_L$ doublets $Q_i^L = (u_i, d_i)^L$ and $L_i^L = (\nu_i, l_i)^L$, while their right-handed partners transform as $SU(2)_L$ singlets. Hypercharges Y of all particles, which correspond to the symmetry $U(1)_Y$, were determined experimentally from the Gell-Mann-Nishijima relation

$$Q = I_{3L} + \frac{Y}{2}, \tag{1.2}$$

where Q is the electric charge which is the same for the left- and right-handed components of the Dirac spinor and I_{3L} is the quantum number associated with the third component of weak isospin. The other (fundamental) constituent of the SM is the spinless *Higgs* boson $\Phi \sim (1,2)_{+\frac{1}{2}}$. It corresponds to the complex $SU(2)_L$ doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{1.3}$$

The Standard Model Lagrangian is usually divided into two main parts: the (highly symmetric) gauge sector and the (symmetry breaking) Higgs sector. The gauge sector is specified by the local symmetry $\mathcal{G}_{\rm SM}$ and by the fermion content (1.1)

$$\mathcal{L}_{\rm SM}^{\rm gauge} = \sum_{i=1..3} \sum_{\psi=Q_i^L..l_i^R} \bar{\psi}_i i\gamma^\mu D_\mu \psi_i - \frac{1}{4} \sum_{\alpha=1..8} G_{\mu\nu}^\alpha G_\alpha^{\mu\nu} - \frac{1}{4} \sum_{a=1..3} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{1.4}$$

where $G_{\mu\nu}^\alpha$, $W_{\mu\nu}^a$ and $B^{\mu\nu}$ are the strengths of the gauge fields of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ respectively. They are given by

$$\begin{aligned} G_{\mu\nu}^\alpha &= \partial_\mu G_\nu^\alpha - \partial_\nu G_\mu^\alpha - g_s f^{\alpha\beta\gamma} G_\mu^\beta G_\mu^\gamma, \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc} W_\mu^b W_\mu^c, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{1.5}$$

and the covariant derivative is given by

$$D_\mu = \left(\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}B_\mu Y\right), \tag{1.6}$$

where g_s, g and g' are coupling constants associated with the three groups of G_{SM} which determine the strengths of the interactions, and n, m run over red, green and blue QCD color states[2]. $\tau^a/2$ are the generators of the $SU(2)_L$ group where τ^a $(a = 1..3)$ are the three Pauli matrices, and $T^\alpha/2$ are the generators of the $SU(3)_C$ group where T^α $(\alpha = 1..8)$ represent the eight Gell-Mann matrices. $f^{\alpha\beta\gamma}$ and ϵ^{abc} in (1.5) are $SU(3)_C$ and $SU(2)_L$ structure constants, respectively. The former are antisymmetric in in index permutation, while the latter are represented by the totally antisymmetric Levi-Civita tensor.

The kinetic part of the Lagrangian for the left-handed quarks is obtained by the minimal coupling where all possible symmetries of the SM are present

$$\mathcal{L}^Q_L = \bar{Q}^L_{n,i}\left\{i\gamma^\mu\left[\left(\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}B_\mu Y\right)\delta_{nm} + ig_s\frac{T^\alpha_{nm}}{2}G^\alpha_\mu\right]\right\}Q^L_{m,i}, \tag{1.7}$$

where i and j are flavor indices and both left-handed quark doublets have the same weak hypercharge $Y = 1/6$. The guage kinetic terms of the Lagrangian for the right-handed quarks, described by $SU(2)_L$ singlets, have the form

$$\mathcal{L}^q_R = \bar{q}^R_{n,i}\left\{i\gamma^\mu\left[\left(\partial_\mu + i\frac{g'}{2}B_\mu Y\right)\delta_{nm} + ig_s\frac{T^\alpha_{nm}}{2}G^\alpha_\mu\right]\right\}q^R_{m,i}, \tag{1.8}$$

where q can be either an up- or a down-type quark. In this case, weak hypercharge and electric charge are the same ($2/3$ for the up-type quarks and $-1/3$ for the down-type quarks).

Due to unknown reasons, only left-handed particles participate in the weak interaction. Left-handed quarks, though, differ from left-handed-leptons in that they take part in

[2]Red, green and blue in the case of the fundamental representation. In the case of the complex conjugate representation, n and m correspond to cyan (antired), magenta (antigreen) and yellow (antiblue).

all known interactions, whereas leptons do not feel the strong interaction, hence the absence of the term responsible for the strong interaction from their Lagrangian

$$\mathcal{L}_L^l = \bar{l}_i^L i\gamma^\mu \left(\partial_\mu + i\frac{g}{2}\tau^a W_\mu^a + i\frac{g'}{2}B_\mu Y\right) l_i^L,$$

and (1.9)

$$\mathcal{L}_R^l = \bar{l}_i^R i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}B_\mu Y\right) l_i^R.$$

l^R here stands for a charged lepton as there is no evidence yet for the existence of right-handed neurinos. If they do exist, they would have zero couplings both to $SU(2)_L$ and to $U(1)_Y$.

Up to this point, all particles are massless. The situation changes, however, when the (local) gauge symmetry of the model breaks down spontaneously as the Higgs field Φ acquires a non-zero vacuum expectation value (*vev*). The Higgs sector contains two terms: the Higgs self-coupling $\mathcal{L}_{\text{SM}}^{\text{Higgs}}$ and the Yukawa Lagrangian $\mathcal{L}_{\text{SM}}^{\text{Yukawa}}$. The latter generates masses for the (electrically) charged fermions, while the former generates masses for the weak gauge bosons. It is given by

$$\mathcal{L}_{\text{SM}}^{\text{Higgs}} = (D_\mu\Phi)^\dagger(D^\mu\Phi) + V(\Phi), \tag{1.10}$$

where the potential term which describes the scalar self-interaction is given by

$$V(\Phi) = \mu^2(\Phi^\dagger\Phi) - \lambda(\Phi^\dagger\Phi)^2. \tag{1.11}$$

The parameters are required to be $\lambda > 0$ and $\mu^2 < 0$. When the Higgs field acquires a non-vanishing *vev*

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.12}$$

with $\langle\Phi\rangle = v = (\sqrt{2}G_F)^{-\frac{1}{2}} \approx 246$ GeV, a spontaneous symmetry breaking (SSB) of the electroweak (EW) group down to the electromagnetic (EM) is triggered

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad \xrightarrow{SSB} \quad SU(3)_C \otimes U(1)_{em}. \tag{1.13}$$

This (*Higgs*) mechanism generates the masses of all gauge particles of the model except for the eight quantum chromodynamis (QCD) gluons and the one qunatum electrodynamics (QED) photon. The scalar kinetic term of (1.10) generates masses for the gauge bosons

$$M^2_{\text{gauge bosons}} \sim \frac{1}{2}\begin{pmatrix} 0 & v \end{pmatrix}\left(\frac{1}{2}g\tau W_\mu + \frac{1}{2}g' B_\mu\right)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.14}$$

where three massive vector bosons $W^\pm_\mu$ and Z_μ appear

$$\begin{aligned} W^\pm_\mu &= \frac{1}{\sqrt{2}}\left(W^1_\mu \mp W^2_\mu\right), \qquad \text{with mass} \qquad M_W = \frac{1}{2}gv, \\ Z_\mu &= \frac{-g'B_\mu + gW^3_\mu}{\sqrt{g^2 + g'^2}}, \qquad \text{with mass} \qquad M_Z = \frac{1}{2}v\sqrt{g + g'^2}, \end{aligned} \tag{1.15}$$

and a fourth vector boson (identified with the photon) remains masseless

$$A_\mu = \frac{gB_\mu + g'W^3_\mu}{\sqrt{g^2 + g'^2}}. \tag{1.16}$$

The coupling constants are related by the weak mixing angle known as the *Weinberg* angle $\theta_{\mathcal{W}}$

$$\sin\theta_{\mathcal{W}} = \frac{g'}{\sqrt{g'^2 + g^2}}, \qquad \cos\theta_{\mathcal{W}} = \frac{g}{\sqrt{g'^2 + g^2}}, \tag{1.17}$$

which was introduced as a parameter that mixes up the B_μ and the W^3_μ bosons into physical states A_μ and Z_μ

$$A_\mu = B_\mu \cos\theta_{\mathcal{W}} - W^3_\mu \sin\theta_{\mathcal{W}}, \qquad Z_\mu = B_\mu \sin\theta_{\mathcal{W}} + W^3_\mu \cos\theta_{\mathcal{W}}. \tag{1.18}$$

From the covariant derivative in Eq. (1.6), the fermion kinetic energy terms can be written in terms of the vector boson mass eigenstates (1.15) and (1.16). They take the form

$$\mathcal{L}_{\rm SM}^{\rm int.} = \bar{Q}_{n,i}^{L}(i\partial)Q_{n,i}^{L}+\bar{q}_{n,i}^{R}(i\partial)q_{n,i}^{R}+\bar{L}_{i}^{L}(i\partial)L_{i}^{L}+\bar{l}_{i}^{R}(i\partial)l_{i}^{R}+g\left(W_{\mu}^{+}J_{W}^{\mu+}+W_{\mu}^{-}J_{W}^{\mu-}+Z_{\mu}J_{Z}^{\mu}\right)+eA_{\mu}J_{em}^{\mu}, \tag{1.19}$$

where e is the coefficient of the electromagnetic interaction

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_{\mathcal{W}}, \tag{1.20}$$

and $J_W^{\mu+}$, $J_W^{\mu-}$, J_W^Z and J_{em} are the charged, neutral and electromagnetic currents, respectively

$$\begin{aligned} J_W^{\mu+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}^L\gamma^\mu l^L + \bar{q}_u^L\gamma^\mu q_d^R\right), \qquad J_W^{\mu-} = \frac{1}{\sqrt{2}}\left(\bar{l}^L\gamma^\mu \nu^L + \bar{q}_d^L\gamma^\mu q_u^R\right), \\ J_\mu^Z &= \frac{1}{\cos\theta_{\mathcal{W}}}\sum_f \bar{f}^L\gamma_\mu\left(I_3^f - Q_f\sin^2\theta_{\mathcal{W}}\right)f^L, \qquad J_{em} = \sum_f Q_f\bar{f}\gamma_\mu f, \end{aligned} \tag{1.21}$$

where Q_f is the electric charge of the fermion f and I_3^f is its quantum number associated with the third component of weak isospin.

Masses of the fermions are generated with the same (Higgs) mechanism. After the SSB, all quarks and (electrically) charged leptons become massive and flavor dynamics arises in. The masses are generated with the Yukawa term of the Higgs sector that describes the interaction of Φ with the fermion fields

$$\mathcal{L}_{\rm SM}^{\rm Yukawa} = Y_d^{ij}\bar{Q}_i^L\Phi d_j^R + Y_u^{ij}\bar{Q}_i^L\tilde{\Phi}u_j^R + Y_l^{ij}\bar{L}_i^L\Phi l_j^R + h.c., \qquad (\tilde{\Phi} = i\tau^2\Phi^\dagger), \tag{1.22}$$

where the Hermitian conjugate of the Higgs Field $\tilde{\Phi} \sim (1,2)_{-\frac{1}{2}}$ is useful in constructing Yukawa interactions invariant under the electroweak group. τ^2 is one of the three Pauli matrices that generate $SU(2)_L$ and Y_f^{ij} are Yukawa couplings. The fermion mass terms for both quarks and charged leptons are then

$$\mathcal{L}_{\rm SM}^{\rm mass} = \bar{u}_i^L M_u^{ij} u_j^R + \bar{d}_i^L M_d^{ij} d_j^R + \bar{l}_i^L M_l^{ij} l_j^R + h.c., \tag{1.23}$$

where M_f^{ij} are 3×3 fermion ($f = u, d, l$) mass matrices which in general are non-diagonal

$$M_f^{ij} = \frac{1}{\sqrt{2}} v Y_f^{ij}. \tag{1.24}$$

The mass spectrum of the fermions exhibits a hierarchy in the Yukawa couplings that increases from one generation to another. Since the fermion mass generation is intimately connected to the scalar sector, this latter, considered the most obscure part of the SM, is the main source of flavor dynamics. Thus, clearly the flavor sector of the SM appears to be one mysterious territory that deserves a deep exploration.

1.1 The flavor sector of the SM

In addition to the local symmetry induced by the gauge structure in Eq. (1.4), a *global* $U(3)^5 = U(3)_q^3 \otimes U(3)_l^2$ flavor symmetry of $\mathcal{L}_{\rm SM}^{\rm gauge}$ also rises from this same structure[3]. Both symmetries get broken with the introduction of the same $SU(2)_L$ scalar doublet Φ: the local symmetry gets spontaneously broken by the ground state *vev* of the Higgs field, while the global flavor symmetry is *explicitly* broken by the Yukawa interaction of Φ with the fermion fields (1.22), as the Yukawa couplings $Y_{u,d,e}$ are in general non-diagonal matrices. In fact, in the absence of Yukawa interactions (i.e. the Yukawa couplings are set to 0), $\mathcal{L}_{\rm SM}$ is just the sum of of covariantized kinetic energy and self-interacting boson terms, where a linear unitary transformation among the fields can be safely made without altering the Lagrangian. Thus, for each of the five SM representations (each is replicated in three copies) (1.1), the redefinition freedom is by 3×3 matrices which are elements of the group $U(3)$. The flavor symmetry group can

[3] $U(3)^5$ flavor symmetry corresponds to the independent unitary rotations of the fermion fields in flavor space, where for each SM representation, the redefinition is by elements of $U(3)$.

SM field	Q_L	u_R	d_R	L_L	l_R
B	$+1/3$	$+1/3$	$+1/3$		
L				$+1$	$+1$
Y	$+1/6$	$+2/3$	$-1/3$	$-1/2$	-1
$U(1)_{pq}$			$+1$		$+1$
$U(1)'$					$+1$

Table 1.1: $U(1)^5$ symmetry quantum numbers assigned to each SM's fermionic field.

be decomposed as[4]

$$G_{flavor} = U(1)^5 \otimes G_q \otimes G_l, \tag{1.25}$$

where

$$G_q = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}, \qquad G_l = SU(3)_{L_L} \otimes SU(3)_{l_R}, \tag{1.26}$$

and the residual flavor symmetry group of the $\mathcal{L}_{\text{SM}}^{\text{gauge}}$, which is not broken by the Yukawa interactions

$$U(1)^5 = U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}} \otimes U(1)_Y \otimes U(1)_{pq} \otimes U(1)', \tag{1.27}$$

where three of the five $U(1)$ subgroups can be identified with the total baryon $\mathcal{B}$ and lepton $\mathcal{L}$ family number conservation, and the weak hypercharge[5] [33]. The two remaining $U(1)$ groups can be identified with U_{pq} (Peccei-Quinn symmetry) where the Higgs has an opposite charge to d^R and l^R fields, and $U(1)'$ which corresponds to a rotation of l^R only. Table (1.1) illustrates the quantum numbers of $U(1)^5$ assigned to each fermionic field. The subgroups that control flavor-changing dynamics and flavor non-universality are the thus the non-Abelian groups G_q and G_l, which are explicitly broken by the Yukawa couplings $Y_{u,d,e}$. The term "*flavor violation*" can thus safely be

[4]Whenever we have a $U(N)$ symmetry, we can alwayas extract a global phase, which is independent of the mixing, and a special unitary transformation $U(N) = U(1) \otimes SU(N)$.

[5]The weak hypercharge Y is gauged and broken only spontaneously by the non-vanishing *vev* of the Higgs field $\langle \phi \rangle \neq 0$.

used to describe processes or parameters that break the $SU(3)^3_q \otimes SU(3)^2_l$ symmetry. To render the mass terms diagonal (i.e. diagonalize the Yukawa matrices) a further linear redefinition of the fields is often adapted. The diagonalization is realized by the introduction of two independent matrices for each Yukawa coupling

$$\lambda_u = V_u^\dagger Y_u W_u, \qquad \lambda_d = V_d^\dagger Y_d W_d, \qquad \lambda_l = U_l^\dagger Y_l W_l, \tag{1.28}$$

where $\lambda_{u,d,l}$ are diagonal, and $V_{u,d}$, U_l and $W_{u,d,l}$ are unitary matrices which relate the interaction (primed) basis and the mass (unprimed) basis via

$$u'^L = V_u u^L, \qquad d'^L = V_d d^L, \qquad L'^L = U_l L^L, \qquad u'^R = W_u u^R, \qquad d'^R = W_d d^R, \qquad l'^R = W_l l^R. \tag{1.29}$$

The invariance of $\mathcal{L}_{\rm SM}^{\rm gauge}$ under G_l allows us to freely choose two matrices U_l and W_l that diagonalize Y_l without breaking gauge invariance, i.e., leading to no phenomenological consequences. This is not the case for the quark sector where the diagonalization of $\mathcal{L}_{\rm SM}^{\rm Yukawa}$ requires one out of two 3×3 matrices that rotate the up and down components of the left-handed quark doublet, plus two unitary matrices that rotate u^R and d^R. In fact

$$\bar{Q}'^L Y_d d'^R \to \bar{Q}'^L (U_Q)^\dagger \left[V_d \lambda_d (W_d)^\dagger\right] (U_d) d'^R, \qquad \bar{Q}'^L Y_u u'^R \to \bar{Q}'^L (U_Q)^\dagger \left[V_u \lambda_u (W_u)^\dagger\right] (U_u) u'^R, \tag{1.30}$$

where U_Q, U_d and U_u are unitary 3×3 elements of G_q group that maintain $\mathcal{L}_{\rm SM}^{\rm gauge}$ invariance. It is clear that U_d can be identified with W_d, and U_u with W_u, but only one of the two V_d and V_u can be identified with U_Q. By convention, we choose the basis where Y_d is diagonal, i.e. $U_Q \equiv V_d$. Equation (1.30) becomes

$$\bar{Q}'^L Y_d d'^R \to \bar{Q}^L \lambda_d d^R, \qquad \bar{Q}'^L Y_u u'^R \to \bar{Q}^L (V_d)^\dagger V_u \lambda_u u^R, \tag{1.31}$$

where

$$Y_d = \lambda_d, \qquad Y_u = V^\dagger \lambda_u, \tag{1.32}$$

and

$$V = V_u^\dagger V_d. \tag{1.33}$$

The diagonal Yukawa matrices are then written as

$$\lambda_d = diag(y_d, y_s, y_b), \qquad \lambda_u = diag(y_u, y_c, y_t), \qquad y_q = \frac{\sqrt{2} m_q}{v}, \tag{1.34}$$

where m_q is the (physical) mass[6] of the q quark. As a result, $\mathcal{L}_{\rm SM}^{\rm gauge}$ is no longer invariant under G_q ($SU(3)_{Q_L}$). This shows, in particular, in the charged-current interaction involving quarks that arises from the term $\bar{Q}_i^L i D Q_i^L$, which is the only term that feels the change of basis. From Eqs. (1.19) and (1.21), we write

$$\mathcal{L}_{\rm SM}^{\rm CC}|_{\rm quarks} = \frac{g}{\sqrt{2}} W_\mu^+ (\bar{q}'^L_{iu} \gamma^\mu q'^L_{id}) + h.c. \quad \overset{q_u, q_d}{} \xrightarrow{\text{mass-basis}} \quad \frac{g}{\sqrt{2}} W_\mu^+ V_{ij} (\bar{q}^L_{iu} \gamma^\mu q^L_{jd}) + h.c. \tag{1.35}$$

Thus, the fact that the flavor symmetry does not allow the diagonalization of both Y_d and Y_u from the left leaves us with a non-trivial unitary mixing matrix V which is nothing but the *Cabibbo-Kobayashi-Maskawa* (CKM) mixing matrix [34, 35] resulting from the different orientations of Y_d and Y_u in $SU(3)_{Q_L}$ group space. It is clear from Eq. (1.35) that $\mathcal{L}_{\rm SM}^{\rm CC}|_{\rm quarks}$, which is diagonal in the flavor (interaction) basis, is no longer diagonal when we switch to the mass basis. Tree-level flavor-changing charged current (FCCC) transitions arise due to the presence of the $V_{\rm CKM}$ matrix. This goes to show its crucial importance in flavor physics, as it is the only source of flavor-changing transitions in the SM.

1.1.1 Cabibbo-Kobayashi-Maskawa (CKM) matrix

The Cabibbo-Kobayashi-Maskawa matrix is a generic 3×3 complex unitary matrix that originates from the Yukawa sector by the miss-alignment of Y_u and Y_d in the $SU(3)_{Q_L}$ subgroup of G_q (flavor space). It depends on three real rotational angles and

[6] $\lambda_u \approx diag\left(6 \times 10^{-6}, 3 \times 10^{-3}, 1\right)$, which is more hierarchical than λ_d.

six complex phases [36]. As a consequence of our choice of the quark basis where Y_d and Y_u have the form in Eq. (1.32), five of the six complex phases can be eliminated (the relative phases of the various quark fields), leaving us with four physical parameters: three real angles and one complex $\mathcal{CP}$-violating phase. Thus, the breaking of the quark flavor symmetry in the SM is controlled by eleven parameters: the six quark masses in $\lambda_{u,d}$ (1.34) and the four parameters of V.

1.1.2 Some properties of the CKM matrix

The standard parametrization of the CKM matrix in terms of its four parameters is [37]

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \tag{1.36}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. θ_{ij} are the three rotational angles ($i, j = 1, 2, 3$) and δ is the complex phase. A strongly hierarchical pattern shows in the off-diagonal elements of the CKM matrix: $|V_{us}|$ and $|V_{cd}|$ values are close to 0.22, $|V_{cb}|$ and $|V_{ts}|$ are of the order 4×10^{-2}, whereas the elements $|V_{ub}|$ and $|V_{td}|$ are of the order 5×10^{-3}. In a more explicit way, this hierarchy[7] is conveniently exhibited in the Wolfenstein parametrization [38], where the matrix elements are expended in powers of the small parameter $\lambda \doteq |V_{us}| \approx 0.22$. However, for the requirement of a sufficient accuracy, the simplest and nowadays commonly adopted parametrization is obtained by a generalization of the Wolfenstein parameters which are defined in terms of the exact parametrization in

[7]The order of magnitude of the V_{CKM} elements is roughly given by $V = \begin{pmatrix} \epsilon^0 & \epsilon^1 & \epsilon^3 \\ \epsilon^1 & \epsilon^0 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \end{pmatrix}$, with $\epsilon \sim 10^{-1}$.

Eq. (1.36)

$$\lambda \doteq s_{12}, \qquad A\lambda^2 \doteq s_{23}, \qquad A\lambda^3(\rho - i\eta) \doteq s_{13}e^{-i\delta}, \tag{1.37}$$

where A, ρ and η are free parameters of order 1. Expanding the Wolfenstein parametrization up to $\mathcal{O}(\lambda^5)$ leads to

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda + \mathcal{O}(\lambda^7) & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 + \mathcal{O}(\lambda^8) \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}, \tag{1.38}$$

where the rescaled variables $\bar{\rho}$ and $\bar{\eta}$ are used

$$\bar{\rho} = \rho(1 - \frac{1}{2}\lambda^2) + \mathcal{O}(\lambda^4), \qquad \bar{\eta} = \eta(1 - \frac{1}{2}\lambda^2) + \mathcal{O}(\lambda^4). \tag{1.39}$$

Due to its unitarity feature, the CKM matrix elements obey the relations

$$\textbf{I)} \sum_{k=1,2,3} V_{ik}^* V_{ki} = 1, \qquad \textbf{II)} \sum_{k=1,2,3} V_{ik}^* V_{kj \neq i}. \tag{1.40}$$

These relations are a distinctive feature of the SM, thus their experimental verification is a powerful consistency check of the model. In fact, relations of type (**I**) suppress the possibility of a fourth family of quarks since the sum is verified experimentally to be very close to 1 [16]. For $i = 1$ and $j = 3$, one of the six relations of type (**II**), implies the relation

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} + 1 = 0 \qquad \longleftrightarrow \qquad [\bar{\rho} + i\bar{\eta}] + [(1 - \bar{\rho}) - i\bar{\eta}] + 1 = 0, \tag{1.41}$$

Which is the most commonly disscussed. It is usually represented in the $(\bar{\rho}, \bar{\eta})$ complex plane as a *unitarity triangle*[8] shown in Fig. (1.1)

The base is of unit length and the internal angles of the triangle are α, β and γ

[8]Sometimes referred to as "fat" 1-3 columns triangle as its sides are of comparable size, unlike the other two triangles.

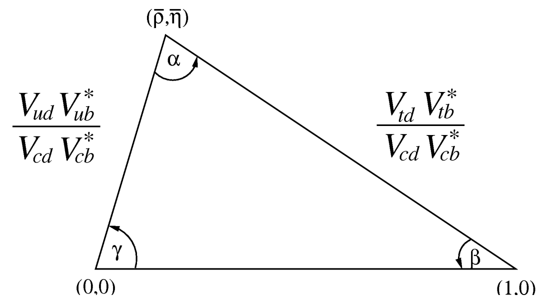

Figure 1.1: The CKM unitarity triangle in the $(\bar{\rho}, \bar{\eta})$ complex plane.

whose senses are indicated as arrows in Fig. (1.1). They are defined as [39]

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}^*V_{ub}^*}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}^*V_{tb}^*}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}^*V_{cb}^*}\right). \tag{1.42}$$

Any phase transformation of the quark fields would not affect Eq. (1.41). As a matter of fact, under such transformation, the triangle in Fig. (1.1) is rotated in the complex plane, and yet it remains intact. i.e. its angles and sides (given by the moduli of the elements of the mixing matrix) remain unchanged. The consistency of Eq. (1.41) can be experimentally tested as both angles and sides of the triangle are observable quantities which can be extracted from suitable experiments [40].

1.1.3 Present status of the CKM fits

The values of λ and A (values of $|V_{us}|$ and $|V_{cb}|$, respectively) are determined with good accuracy from the $K \longrightarrow \pi l \nu$ and $B \longrightarrow X_c l \nu$ decays. Their numerical values are determined with good accuracy [41]

$$\lambda = 0.22500 \pm 0.00100, \qquad A = 0.826 \pm 0.012. \tag{1.43}$$

Thus, all the observables sensitive to the CKM matrix elements can be expressed as constraints on the remaining parameters $\bar{\rho}$ and $\bar{\eta}$. Fig. (1.2) shows that the resulting

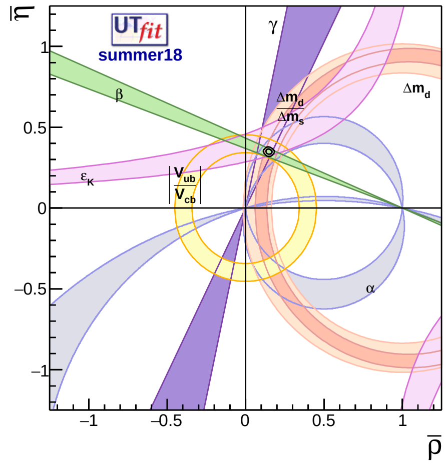

Figure 1.2: Allowed region in the $\bar{\rho}$, $\bar{\eta}$ plane as obtained by the UTfit collaboration.

constraints are all consistent with a unique values of $\bar{\rho}$ and $\bar{\eta}$ [41]

$$\bar{\rho} = 0.148 \pm 0.013, \qquad \bar{\eta} = 0.348 \pm 0.010. \tag{1.44}$$

The consistency of the constraints shown in Fig. (1.2) testifies for the consistency of the SM in describing flavor physics. Therefore, qualitatively speaking, little room is left for non-SM contributions in flavor changing transitions. In the SM, FCCC arise already at tree-level, whereas, FCNC processes are highly suppressed. In fact, not only they arise at one loop, but also these transitions always involve at least one off-diagonal element of the CKM matrix and are further suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism [42]. This makes these transitions golden channels to look for NP effects. Similarly to the quark sector, leptons would also mix. In fact, the unitary *Pontecorvo-Maki-Nakagawa-Sakata* (PMNS) mixing matrix would arise when we switch from the flavor (interaction) basis to the the mass basis in the lepton sector. However, as the SM predicts massless neutrinos, the U_{PMNS} is not part of the SM. It would be significant, however, in other BSM scenarios, as in the case we are adopting (see Section (3.69)).

1.2 Lepton universality (LU)

Being based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, the SM is structured in a certain way that makes the three flavors of the fermion fields have the same gauge charge assignements, which leads to the same structure of their couplings to the model's gauge bosons (g_s, g, g') [43]. This feature, known as *universality*, holds even when the SM's gauge symmetry gets broken down spontaneously as the Higgs field acquires a non-vanishing *vev*. The Higgs mechanism, however, leads to the only difference there is between the three families. In fact, the diagonalisation of the mass matrices, generated by the Yukawa interactions between the Higgs and the fermion fields, yields mixing terms between weak (interaction) and mass (physical) eigenstates, namely the CKM and the PMNS matrices, that occur in the coupling of the weak gauge bosons to quarks and leptons, respectively. Within the SM, as this latter does not account for the neutrino mass, the PMNS matrix plays no role. As a consequence, in the case of the (tree-level) FCCC transition $b \longrightarrow cl^- \bar{\nu}_l$ for instance, a single (experimentally determined) CKM matrix element V_{cb} is involved for all processes of the sort, regardless of what the flavor of l might be. As for the (loop-level) FCNC transition $b \longrightarrow sl^+l^-$, the matrix elements $V_{ib}V_{is}^*$, which are involved in the LU testing observable, depend on the flavor of the up-type quark running in the loop $(i \equiv u, c, t)$. Due to the unitarity of the CKM matrix and its hierarchical structure, all of its elements can be expressed in terms of a leading term $V_{tb}V_{ts}^*$ and a Cabibbo-suppressed contribution $V_{ub}V_{us}^*$. With that being said, even with the richness of the flavor structure that appears in the quark sector, the leptonic sector is easier to analyze. In fact, in order to determine accurately the mixing parameters of quarks, we need to establish a good understanding of hadronization effects in flavor-changing (FC) transitions as quarks are confined within hadrons. Leptons would also mix had neutrinos have non-vanishing masses. However, as the neutrino mass eigenstates cannot be distinguished experimentally[9], it is insignificant for the phenomenology. Thus, a sum over the amplitudes associated to the production of all

[9]Neutrino mass mass differences are negligible compared to other scales and they are not detected in experiments.

three possible (anti)neutrino mass eigenstates is required. For the decay width of the FCCC transition $b \longrightarrow c l^- \bar{\nu}_l$, for instance, because the (anti)neutrino mass eigenstate is unspecified, the decay width features a factor of the form $\sum_{i=1,2,3} |U_{li}|^2$, where U_{li} is the PMNS matrix element describing the overlap of each (anti)neutrino mass eigenstate i with the produced lepton l. Due to its unitarity in the SM, the sum of the PMNS matrix elements above should be equal to 1. Therefore, it is ignored in most computations. Moreover, due to the absence of a direct lepton-gluon vertex, the leptonic sector provides an easier subject to obtain precise theoretical predictions which can be compared with the available data. In fact, semi-leptonic transitions, such as $\tau^- \longrightarrow \nu_\tau M^-$ or $M^- \longrightarrow l^- \bar{\nu}_l$ $(M = \pi, K)$ provide accurate tests of the leptonic couplings even with the presence of hadronization, which only involves gluonic exchanges between the quarks of a single hadronic current. Yet, to a good approximation, QCD effects would cancel out had we taken appropriate ratios of different semi-leptonic transitions with identical hadronic components. This what makes either purely leptonic or semileptonic processes that involve leptons of different generations but with the same quark transition, preferable to test LU.

1.3 Lepton universality tests

Throughout the years, lepton universality has been tested using a variety of probes including the production and the decay of the electroweak gauge bosons, purely leptonic and semi-leptonic decays of mesons and the decay of quarkonia. While no significant deviations from the SM expectation have been observed in several flavor-changing transitions, measurements from experiments of semi-leptonic B decays at the B-*factories* (Belle, BaBar, Belle-II) as well as LHCb, hinted at a possible violation of LU, which would be an unambiguous sign of the existence of physics beyond the SM. If NP originates at a scale Λ in the TeV range, then its effects on weak B decays would be suppressed by inverse powers of Λ. Therefore, NP should be looked for in either processes which are suppressed or forbidden (hidden) within the SM, or in observables that are predicted with high precision in the SM. The SM predictions for theses processes can be computed by seperating short-and long distance contributions through an *effective Hamiltonian approach*, when branching ratios of multiple decays are compared.

1.3.1 Lepton universality tests beyond the B sector

Several experiments testing LU, in other sectors of the SM, have exhibited an agreement with the SM expectation meaning that no LU violations are present in the according flavor changing processes.

Electroweak sector

A large number of experiments have proved that the three lepton families have the same behavior when it comes to their coupling with the electroweak bosons (charged W and neutral Z). For instance, experiments running at e^+e^- colliders[10], at $p\bar{p}$ (Tevatron) and at pp (LHC) have obtained the most precise results. Measurements of $Z \longrightarrow l^+l^-$ ($l \equiv e, \mu, \tau$) partial widths are shown to agree well among each other [16]. In fact, the leptonic partial-widths ratios $\Gamma_{Z\longrightarrow l^+l^-}/\Gamma_{Z\longrightarrow e^+e^-}$, where $l \equiv \mu, \tau$ have testified for LU in Z decays as they turned out to be equal [44, 45], agreeing thus with the SM prediction. Ratios comparing the decays $W^- \longrightarrow e^-\bar{\nu}_e$ and $W^- \longrightarrow \mu^-\bar{\nu}_\mu$ which depend on $(g_e/g_\mu)^2$ with g_l being the coupling strength of $W^- \longrightarrow l^-\bar{\nu}_l$ are also shown to be in good agreement with LU [46].

Purely leptonic decays

Pure leptonic decays of the τ lepton can also be used to test LU in FCCC. For the decay modes[11] $\tau^- \longrightarrow e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \longrightarrow \mu^-\bar{\nu}_\mu\nu_\tau$, the most stringent experimental tests obtained on the universality of the charged-current couplings to leptons (g_l) implies

[10] LEP and SLC running at the Z pole or LEP2 where the direct production of W boson pairs is enabled by the center-of-mass energy.

[11] Other kinematically allowed final states in the τ lepton decay are the semileptonic channels $\tau^- \longrightarrow \nu_\tau d\bar{u}$ and $\tau^- \longrightarrow \nu_\tau s\bar{u}$.

that the first and the second lepton families are universal at the 0.14% level [16, 46]. Moreover, from the combination of the precise measurement of the branching fraction $\tau^- \longrightarrow e^-\bar{\nu}_e\nu_\tau$ and of the τ and μ lifetimes [43], the ratios of the FCCC couplings g_τ/g_μ and g_τ/g_e are found to be 1.0011 ± 0.0015 and 1.0030 ± 0.0015, respectively [46] which both represent the most stringent experimental LU tests available today involving the couplings of the first/second lepton family to the third one.

Semileptonic decays

Leptonic decays of pseudoscalar mesons $M^- \longrightarrow l^-\bar{\nu}_l$, the semileptonic $\tau^- \longrightarrow \nu_\tau M^-$ decay channels ($M = \pi, K$) and the leptonic decays of quarkonia also serve as powerful tests of LU. Leptonic decays of charged pions or kaons provide the most stringent constraints. In fact, the ratio of the partial decay widths $\Gamma_{K^-\longrightarrow e^-\bar{\nu}_e}/\Gamma_{K^-\longrightarrow \mu^-\bar{\nu}_\mu}$ is precisely computed within the SM to be $(2.477 \pm 0.001) \times 10^{-5}$ [47]. This ratio is now precisely known from several experiments that were dedicated to its measurement. The world average has shown to agree with the SM expectation [16]. For charged pions, the ratio $\Gamma_{\pi^-\longrightarrow e^-\bar{\nu}_e}/\Gamma_{\pi^-\longrightarrow \mu^-\bar{\nu}_\mu}$ has also been measured [16]. These measurements testing the coupling of the W bososn to the first two families of leptons $(g_e/g_\mu)^2$, have shown to be consistent with—but one order of magnitude less precise than—the SM prediction [47]. The ratio $\Gamma_{J/\psi\longrightarrow e^+e^-}/\Gamma_{J/\psi\longrightarrow \mu^+\mu^-}$ also provides an accurate test of LU with a precision of 0.31% [16]. Other quarkonia (e.g. $\psi(2S)$ and $\Upsilon(2S)$) leptonic decay measurements yield constraints that are less precise by an order of magnitude. The charmed-meson sector also enables a powerful probe of LU as the ratio of the partial decay widths $\Gamma_{D_s^-\longrightarrow \tau^-\bar{\nu}_\tau}/\Gamma_{D_s^-\longrightarrow \mu^-\bar{\nu}_\mu}$ has shown to agree also with the SM prediction [48, 49].

Semi-leptonic transitions, such as $K \longrightarrow \pi l^-\bar{\nu}_l$ and $D \longrightarrow K l^-\bar{\nu}_l$, can also serve as tests of LU in FCCC. However, in order to be competitive with the leptonic decays, these tests require the knowledge of the ratio of the scalar and vector form factors f_0/f_+ with a very high level of accuracy. This problem is not encountered in leptonic decays as the main hadronic input (meson decay constants) cancels out of the LU ratios.

LU tests in FCNC semi-leptonic decays such as $K \longrightarrow \pi l^+ l^-$, $D \longrightarrow \pi(\rho) l^+ l^-$ or $D_s^- \longrightarrow K^-(K^{*-}) l^+ l^-$ also lack accuracy as these decay modes are dominated by long-distance hadronic contributions that are very difficult to estimate theoretically [50, 51].

1.3.2 Current status of the B anomalies

Experiments investigating the B anomalies have been carried out at the LHC and the two B-factories[12]: the one at SLAC National Laboratory in California and the other one at KEK in Japan. The first comprised of the PEP-II collider [52] and the BaBar detector [53], which completed taking data in 2008[13], while the second comprised of the KEKB collider [54] and Belle experiment [55], which has ceased operating in 2012 and is now upgraded to Belle II [56]. It has started collecting data in 2018. As for the LHC, three major experiments are concerned with the study of B physics, namely ATLAS, CMS and LHCb. This latter was designed to specifically study the production and the decay of b and c hadrons.

A class of interesting B-physics observables which consitutes a powerful LFU test is given by R-ratios which are ratios of branching fractions of semi-leptonic decays with different lepton flavors in the final states, predicted with high precision in the SM. As the hadronic uncertainties affecting the individual branching fractions (hadronic form-factors) cancel out in the ratio, R-ratios are very clean observables [57]. In particular, according to the underlying quark transition, the ratios of interest reported by LHCb, can be grouped into two categories: ones that assess deviations from τ/l universality in $b \to c l \bar{\nu}_l$ charged currents [58], and the ones that assess deviations from μ/e universality

[12] B-factories are asymmetric e^+e^- colliders built with purpose of producing a huge number of B mesons. They operated at the $\Upsilon(4S)$ resonnace which decays immediately into a $B\bar{B}$ pair.

[13] It was supposed to be followed by the SuperB experiment, to be built at the Cabbibo Laboratory in Italy, but was canceled by the Italian government in 2012.

in $b \rightarrow s l^+ l^-$ neutral currents. They are defined, respectively, as

$$R^{\tau/l}_{D^{(*)}} = \frac{\mathcal{B}\left(B \longrightarrow D^{(*)}\tau\bar{\nu}_\tau\right)}{\mathcal{B}\left(B \longrightarrow D^{(*)}l\bar{\nu}_l\right)},\ [l = e, \mu], \qquad R^{\mu/e}_{K^{(*)}_{[q^2_{min}, q^2_{max}]}} = \frac{\mathcal{B}\left(B \longrightarrow K^{(*)}\mu^+\mu^-\right)_{q^2 \in [q^2_{min}, q^2_{max}]}}{\mathcal{B}\left(B \longrightarrow K^{(*)}e^+e^-\right)_{q^2 \in [q^2_{min}, q^2_{max}]}}, \tag{1.45}$$

where $R_{K^{(*)}}$ are measured over specific ranges for the squared dilepton invariant mass q^2 (in GeV^2/c^4). In the SM, both R-ratios are expected to be unity due to the fact that the weak interactions are lepton-flavor universal, i.e. gauge bosons couple in the same way to all lepton flavors. The only sources of difference between the generations are the masses of the charged leptons and their couplings to the Higgs boson, which have a negligible effect on R-ratios. It has been observed that QED corrections could induce up to 10% in $R_{K^{(*)}}$, although, the analysis performed in Ref. [59] had shown explicitly that these corrections do not exceed ~ 0.03 in the region $1\ GeV^2 < q^2 < 6\ GeV^2$. Therefore, any deviation from unity of $R_{K^{(*)}}$ in this region would constitute a clear signal of NP.

The statistically most significant data that point towards LFUV in both charged and neutral-current transitions are

$$\begin{aligned} R^{\mu/e}_{K^{(*)}} \mid_{q^2 \in [q^2_{min}, q^2_{max}]} &= \begin{cases} R_{K[1.1,6.0]} = 0.846^{+0.060+0.016}_{-0.054-0.014} \quad [60], & 2.5\sigma \\ R_{K^*[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03 \quad [61], & 2.7\sigma \ , \\ R_{K^*[1.1,6.0]} = 0.69^{+0.11}_{-0.07} \pm 0.03 \quad [61], & 3.0\sigma \end{cases} \\ R^{\tau/l}_{D^{(*)}} &= 0.293 \pm 0.038 \pm 0.015, \qquad R^{\tau/l}_{D} = 0.375 \pm 0.064 \pm 0.026, \end{aligned} \tag{1.46}$$

where the values[14] for the CC anomaly $R^{\tau/l}_{D^{(*)}}$ follow from the average [48] of LHCb, Belle and BaBar data [58,62,63]. Furthermore, recent experimental results have shown that the R-ratios are not the only tensions in semi-leptonic B dacays. In fact, LHCb has reported a strong evidence for a deviation from the SM observed in the angular distribution of the decay products in the decays $B^0 \longrightarrow K^{*0}\mu^+\mu^-$. The most prominent deviation concerns the angular distribution of $B \longrightarrow K^*\mu^+\mu^-$, which shows in the

[14]The first errors are statistical and the second ones are systematic.

observable P'^5 that exhibits a 3σ deviation from what is expected in the SM [64, 65]. Additional tensions also arise in the branching ratios $\mathcal{B}(B \longrightarrow K^*\mu^+\mu^-)$ and $\mathcal{B}(B \longrightarrow \phi\mu^+\mu^-)$ [66, 67].

In terms of NP, B anomalies are investigated by means of two practical approaches that allow the description of NP contribution to observables: one that relies on a specific BSM theory while the other approach is based on an effective Hamiltonian that separates long- an short-distance contributions.

Chapter 2

Theoretical treatment of the B anomalies

Despite its impressive phenomenological success in flavor and electroweak physics, many convincing arguments motivate us to believe that the SM is not a complete theory, but rather a low-energy limit of a more complete one to which the extension is still not clear. The completion is believed to be achieved with the addition of some new (heavy) particles that would exist at a higher energy scale. The new degrees of freedom that are assumed to complete the theory are either incorporated within an explicit BSM scenario, or described in a model-independent way. Within the former approach, the SM's gauge group is enlarged to a broader one and gets recovered when the extended gauge symmetry breaks down spontaneously at a higher energy scale, leading to massive gauge particles that, supposedly, mediate the interaction. Within the latter, on the other hand, the new degrees of freedom get integrated out as they are heavier than the SM particles, and their effects are encoded in dimensionless coefficients that couple to local operators. As a consequence, physics beyond the SM, as well as within (SMEFT) [68], is described by means of an *effective field theory* (EFT) approach where the SM Lagrangian becomes the renormalizable part of a more general local Lagrangian made up by a series of local operators of dimension $d > 4$. These operators are constructed in terms of SM fields and are suppressed by inverse powers of an effective scale Λ which parametrizes the mass scale of the new heavy degrees of freedom that are integrated out ($\Lambda > m_W$). To investigate the B anomalies, practical tools are needed to describe NP contributions to semi-leptonic B decays. Rather than building an explicit extension of the SM (model-dependent approach), the effective Hamiltonian approach (model-independent approach) can be applied to both FCCC and FCNC decays.

2.1 Effective field theories (EFTs)

For processes whose typical energy E lies far below the energy scale of the interaction responsible for the process—which is usually set by the mass M of the (heavy) mediator—we naturally tend to separate the effects coming from different scales by means of an EFT approach, where the massive degrees of freedom are integrated out, and their effects are encoded in short-distance coefficients multiplying operators built from light fields. The idea is similar to building the Fermi theory within the SM starting from the electroweak theory. The Former—valid for β decays—is adapted for low-energy transitions involving only light particles where the propagation of the massive (and/or energetic) W gauge boson is neglected when integrated out. This leads to a point-like four-fermion interaction and the effects of the massive degree of freedom are absorbed into the short-distance coefficient G_F[1], called the Fermi coupling constant, that weights the contribution of the long-distance propagating operators built from light fields.

Problems involving different energy scales are usually dealt with by means of two main theoretical tools: *operator product expansion* (OPE) and *renormalization group equations* (RGE). The OPE technique consists of building an effective Lagrangian by a series of operators of dimension $d > 4$. For the purpose of computing the amplitude of a (decay) process, the full theory below M can be replaced by

$$\mathcal{L}_{\text{eff}} = \sum_{d>4} \sum_{n=1}^{N_d} \frac{\mathcal{C}_n^{(d)}}{M^{d-4}} \, \mathcal{O}_n^{(d)}(\text{light fields}), \tag{2.1}$$

[1] $G_F/\sqrt{2} = g^2/8M_W^2$.

where the $\mathcal{O}_n^{(d)}$ are all the possible d-dimensional operators (N_d) compatible with the symmetry of the theory built with fields lighter that M. $\mathcal{C}_n^{(d)}$ is the (dimensionless) *Wilson coefficient* that encodes the effects of the removed (heavy) mediator. Since the operators of higher dimensions can usually be neglected in weak decays[2], the series can safely be truncated at $d = 6$. For the purpose of making up the effective Lagrangian $\mathcal{L}_{\text{eff}}$ that reproduces the meson decay amplitude $\mathcal{A}$—supposedly known and computed within the full theory ($\mathcal{L}_{\text{full}}$) at a given order μ—, the Wilson coefficients $\mathcal{C}_n$ are determined by *matching* both theories when requiring

$$\mathcal{A}_{\text{full}}(I \longrightarrow F) = \mathcal{A}_{\text{eff}}(I \longrightarrow F) = \frac{1}{M^2} \sum_n \mathcal{C}_n(\mu) \ \langle F|\mathcal{O}_n(\mu)|I\rangle. \tag{2.2}$$

For the one-loop case, Wilson coefficients are found by means of the equation

$$\mathcal{C}_n(\mu) = \mathcal{C}_n(M) + k\frac{\alpha}{4\pi}\ln\frac{M}{\mu}, \tag{2.3}$$

where $\mathcal{C}_n(M)$ is the Wilson coefficient we would find by doing a matching at the tree-level (initial conditions), α is the strength of the interaction that originates one-loop corrections and k is some constant. This procedure amounts to computing the Wilson coefficients in ordinary perturbation theory. To evaluate the Wilson coefficients at the energy scale of the process we are interested in, a simple substitution $\mu = E$ in Eq. (2.3) would not be the right way to go since a large logarithm $\alpha \ln \dfrac{M}{E}$ might arise due to the large gap between both energy scales. As a consequence, ordinary perturbation theory would no longer make sense as it breaks down ($\alpha \ln \dfrac{M}{E}$ would no longer serve as a good expansion parameter). This issue can be solved by switching to *RG-improved perturbation theory*, which amounts to fixing the divergences observed in the matrix elements $\langle \mathcal{O}_n \rangle$ by means of *operator renormalization* [70]. We define the

[2]Operator of higher dimensions correspond to terms of the order $\mathcal{O}\left(\dfrac{k^2}{M_W^2}\right)$ where k, the momentum transfer through the W propagator is small compared to M_W [69].

renormalization matrix $Z^{\mathcal{O}}$ as[3]

$$\mathcal{O}_n^{(0)}(q) = \left(Z_{nm}^{\mathcal{O}}\right)^{-1} \mathcal{O}_m(q), \tag{2.4}$$

which relates the unrenormalized ($\mathcal{O}_n^{(0)}$) and the normalized ($\mathcal{O}_m$) operators. The bare effective Lagrangian (where fields and couplings are considered as bare quantities) can be rewritten as

$$\begin{aligned}\mathcal{L}_{\text{eff}} &\propto \mathcal{C}_n \mathcal{O}_n^{(0)}(q^{(0)}) \\ &\equiv Z_q^{(1/2)^x} \left(Z_{nm}^{\mathcal{O}}\right)^{-1} \mathcal{C}_n \mathcal{O}_m(q) \\ &\equiv \mathcal{C}_n \mathcal{O}_n + \left(Z_q^{(1/2)^x} \left(Z_{nm}^{\mathcal{O}}\right)^{-1} - \delta_{nm}\right) \mathcal{C}_n \mathcal{O}_m \\ &\equiv \mathcal{C}_n \mathcal{O}_n + \left(Z_{nm}^{-1} - \delta_{nm}\right) \mathcal{C}_n \mathcal{O}_m,\end{aligned} \tag{2.5}$$

where x is the number of the external quark fields which are renormalized according to

$$q^{(0)} = Z_q^{1/2} q. \tag{2.6}$$

Alternatively, Wilson coefficients are renormalized using

$$\mathcal{C}_n^{(0)} = Z_{nm}^{\mathcal{C}} \mathcal{C}_m, \qquad \text{with} \qquad Z_{nm}^{\mathcal{C}} = \left(Z_{mn}\right)^{-1}. \tag{2.7}$$

The bare Lagrangian is thus expressed in terms of renormalized fields and coefficients with the addition of *countertems*. The renormalization matrix Z_{nm} is obtained by requiring that the corresponding counterterm cancels the divergences in $\mathcal{A}_{\text{eff}}$ (the matrix elements $\left\langle \mathcal{O}_n^{(0)} \right\rangle$). Namely by demanding

$$\left\langle \mathcal{O}_n^{(0)} \right\rangle = \left(Z_{nm}\right)^{-1} \left\langle \mathcal{O}_m \right\rangle, \tag{2.8}$$

[3]The renormalization constant Z_{nm} is a matrix so that operators that carry the same quantum numbers can *mix under renormalization*.

where Z_{nm} relates the unrenormalizd $\left\langle \mathcal{O}_n^{(0)} \right\rangle$ and the renormalized $\langle \mathcal{O}_m \rangle$ amputated Green functions. We define the anomalous dimension matrix for the operators as

$$\gamma = Z^{-1} \frac{d}{d\mu} Z. \tag{2.9}$$

Then, by demanding a non-dependence of the amplitude $\mathcal{A}$ of μ, we find that the $\mathcal{C}_n$ obey the following series of *renormalization group equations* RGE whose structure is fully determined by the anomalous dimensions of the effective operators.

$$\mu \frac{d}{d\mu} \mathcal{A} = 0 \;\Rightarrow\; \mu \frac{d}{d\mu} \mathcal{C}_n(\mu) = \gamma^T_{nm} \mathcal{C}_m(\mu). \tag{2.10}$$

It is solved, formally, in terms of an evolution matrix U as

$$\mathcal{C}_n(\mu) = U(\mu, M)_{nm} \mathcal{C}_m(M), \tag{2.11}$$

with $U(\mu, M)$ is a matrix that describes the evolution of the Wilson coefficients from the high-energy scale M down to the appropriate low energy scale μ [70]. In practice, the problem reduces to solving the RGE (2.10) using the initial condition $\mathcal{C}_n(M)$ (2.3) which is obtained by the matching procedure at high energy $\mathcal{O}(M_W)$ where the substitution $\mu = M$ avoids large logarithms and thus allows the use of ordinary perturbation theory. The solution will then be ran down to the appropriate energy scale μ with the help of RGE. The first step is where NP may appear as the Wilson coefficients $\mathcal{C}_n(M)$ would be modified had NP been heavy.

Within the SM, effective field theories are widely employed, despite the fact that full theory is known. As a matter of fact, the approach has the advantage of making the processes' amplitudes easier to compute. The full potential of EFTs show when it comes to accounting for higher order corrections, e.g. QCD corrections in non-leptonic transitions (see Appendix (A)).

2.1.1 Effective Lagrangian for b-quark decays.

Weak B decays can also be analyzed by means of an EFT approach both within and beyond the SM, since their typical energy $E \sim m_b$ lies far below the EW scale $m_{\rm EW}$[4] as well as the NP scale Λ. Within the SM, low-energy transitions involving only light particles are described by dimension-six four-fermion operators where W and Z are not considered as dynamical degrees of freedom; their presence is taken into account in the Wilson coefficient G_F in this case. The effective Lagrangian used to compute tree-level amplitudes is the well-known Fermi Lagrangian

$$\begin{aligned} \mathcal{L}_{\rm eff}^{\rm SM} &= -\frac{4G_F}{\sqrt{2}}(J_Z^\mu J_{\mu,Z} + J_W^{\mu+} J_{\mu-,W}) \\ &= -\frac{4G_F}{\sqrt{2}} \sum_n \mathcal{C}_n \mathcal{O}_n \end{aligned} \tag{2.12}$$

where the neutral and charged currents are defined in Eq. (1.21) and $\mathcal{C}_n = 1$ at the electroweak scale. When applying this procedure to physics beyond the SM, we consider this latter as the renormalizable part of an effective theory obtained by integrating out heavy degrees of freedom arising at the high scale ($\Lambda \gg m_{\rm EW}$) at which NP is assumed to originate. The full theory in the energy window $m_{\rm EW} \ll {\rm E} \ll \Lambda$ can then be replaced by an effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{\rm SM} + \mathcal{L}_{\rm eff}^{\rm NP}, \tag{2.13}$$

where $\mathcal{L}_{\rm eff}^{\rm NP}$ is the effective Lagrangian that describes NP. It should be invariant under the SM gauge group and should contain only SM particles. It takes the form

$$\mathcal{L}_{\rm eff}^{\rm NP} = \frac{1}{\Lambda} \sum_n \mathcal{C}^{(5)} \mathcal{O}_n^{(5)} + \frac{1}{\Lambda^2} \sum_n \mathcal{C}^{(6)} \mathcal{O}_n^{(6)}. \tag{2.14}$$

[4] $m_{\rm EW}$ identifies m_t, $m_{W,Z}$ and $\frac{v}{\sqrt{2}}$.

The superscripts (5) and (6) denote five[5] and six-dimensional operators[6]. Higher dimensional operators ($d \geq 7$) can safely be neglected and the series can be truncated at $d = 6$ [69]. Hereafter, the six-dimensional (four-fermion) operators will be denoted by $\mathcal{O}_n$. Wilson coefficients $\mathcal{C}_n$ are determined in perturbation theory by matching the full theory onto the effective one at the EW scale to compute the initial conditions $\mathcal{C}_n(m_{EW})$ (2.3). After computing the operator anomalous dimension matrix, RGE are used to describe the evolution of the C_n down to the low-energy scale.
When applying this procedure to physics beyond the SM, the nature of the degrees of freedom integrated out is not known and so are the values of the effective couplings of the higher-dimensional operators. Nevertheless, this approach allows us to analyze all possible realistic extensions in terms a limited number of parameters (the Wilson coefficients). For weak B decays, the energy of the process $E \sim m_b$ is much smaller than the energy scale of the interaction responsible for the process set by the mass M of the corresponding mediator (EW as well as the NP scale $\sim$ TeV), thus, B decays can be analyzed by means of an EFT approach.

An effective approach to FCNC b-quark decays

B-decay anomalies $R^{\mu/e}_{K^{(*)}}$ (1.45) are based on the neutral-current transition $B \longrightarrow K^{(*)}l^+l^-$, ($l = \mu, e$). At the quark-level, $b \longrightarrow s$ transitions are described within the SM by the effective Lagrangian that contains the operators which contribute to the semi-leptonic decay modes $b \longrightarrow sl^+l^-$, $b \longrightarrow s\nu\bar{\nu}$ and the radiative process $b \longrightarrow s\gamma$ [57, 70, 71]

$$\mathcal{L}^{\text{NC,SM}}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left(\lambda^u_{bs} \sum_{n=1}^{2} \mathcal{C}_n \mathcal{O}^u_n + \lambda^c_{bs} \sum_{n=1}^{2} \mathcal{C}_n \mathcal{O}^c_n - \lambda^t_{bs} \sum_{n=3}^{10} \mathcal{C}_n \mathcal{O}_n - \lambda^t_{bs} \mathcal{C}_\nu \mathcal{O}_\nu + h.c. \right), \tag{2.15}$$

where $\lambda^p_{bs} \equiv V_{pb} V^*_{ps}$ (p stands for the up-type quark) and $\mathcal{O}_n$ denote the different types of operators: the current-current operators $\mathcal{O}^{u,c}_{1,2}$ and the QCD penguin four-quark op-

[5]Weinberg operator is the only five-dimensional operator. It is responsible for the generation of neutrino masses. Since it does not play any role in our analysis, we safely neglect it.

[6]Out of the whole set of six-dimensional operators [72, 73], we will focus on four-fermion operators.

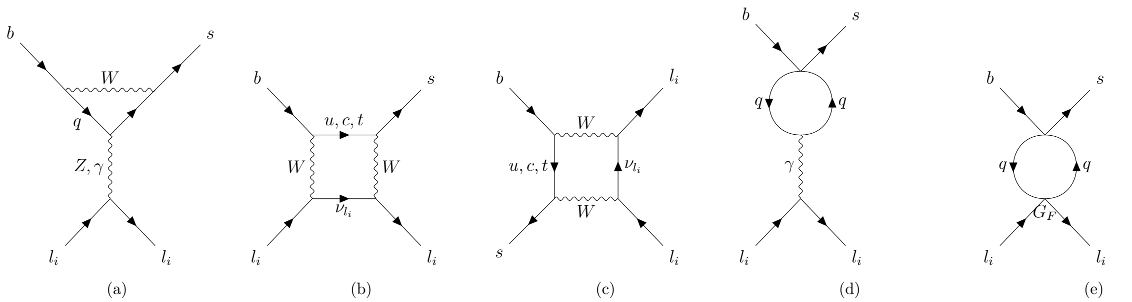

Figure 2.1: Diagrams relevant for the computation of $\mathcal{C}_9$ and $\mathcal{C}_{10}$ in the full theory (a)-(c) and in the effective theory (d), (e).

erators $\mathcal{O}_{3-6}$ (where we sum over $q = u, d, s, c, b$), the electromagnetic $\mathcal{O}_7$ and chromo-magnetic dipole operator $\mathcal{O}_8$ and the semi-leptonic operators $\mathcal{O}_9$, $\mathcal{O}_{10}$ and $\mathcal{O}_\nu$ [74]

$$
\begin{aligned}
&\mathcal{O}_1^p = (\bar{s}_L\gamma_\mu T^\alpha p_L)\,(\bar{p}_L\gamma^\mu T^\alpha p_L)\,, && \mathcal{O}_2^p = (\bar{s}_L\gamma_\mu p_L)\,(\bar{p}_L\gamma^\mu b_L)\,,\\
&\mathcal{O}_3 = (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q(\bar{q}\gamma^\mu q)\,, && \mathcal{O}_4 = (\bar{s}_L\gamma_\mu T^\alpha b_L)\textstyle\sum_q(\bar{q}\gamma^\mu T^\alpha q)\,,\\
&\mathcal{O}_5 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho q)\,, && \mathcal{O}_6 = (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho T^\alpha b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T^\alpha q)\,,\\
&\mathcal{O}_7 = \frac{e}{16\pi^2}m_b\,(\bar{s}_L\sigma^{\mu\nu}b_R)\,F_{\mu\nu}, && \mathcal{O}_8 = \frac{g_s}{16\pi^2}m_b\,(\bar{s}_L\sigma^{\mu\nu}T^\alpha b_R)\,G^\alpha_{\mu\nu},\\
&\mathcal{O}_9 = \frac{e^2}{16\pi^2}\,(\bar{s}_L\gamma^\mu b_L)\left(\bar{l}_i\gamma_\mu l_i\right), && \mathcal{O}_{10} = \frac{e^2}{16\pi^2}\,(\bar{s}_L\gamma^\mu b_L)\left(\bar{l}_i\gamma_\mu\gamma_5 l_i\right),\\
&\mathcal{O}_\nu = \frac{e^2}{16\pi^2}\,(\bar{s}_L\gamma^\mu b_L)\,(\bar{\nu}_i\gamma_\mu(1-\gamma_5)\nu_i)\,,
\end{aligned}
\tag{2.16}
$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and T^α are the $SU(3)$ color matrices. The sum over repeated flavor indices in semi-leptonic operators is understood. For the semi-leptonic process $B \longrightarrow K l^+ l^-$ the relevant operators involve either charged leptons or neutrinos. They are $\mathcal{O}_9$, $\mathcal{O}_{10}$ and $\mathcal{O}_\nu$. The effective Lagrangian that describes the $R_K^{\mu/e}$ anomalies reads

$$
\mathcal{L}_{\text{eff}}^{\text{NC,SM}} = -\frac{4G_F}{\sqrt{2}}\lambda^t_{bs}\left(\mathcal{C}_9\mathcal{O}_9 + \mathcal{C}_{10}\mathcal{O}_{10} + \mathcal{C}_\nu\mathcal{O}_\nu\right), \tag{2.17}
$$

where $\mathcal{O}_\nu$ is the operator that contributes to $B \longrightarrow K\bar{\nu}\nu$. The Feynman diagrams entering the computation of $\mathcal{C}_9$ and $\mathcal{C}_{10}$ at the matching scale m_{EW} both in the full and in the effective theory are displayed in figure 2.1.

Essentially, the low-energy framework is effected by NP when this latter's new heavy degrees of freedom modify the Lagrangian (2.15). In fact, NP presence can violate both the lepton flavor universality and the lepton flavor as it will contribute with a non-universal and a non-diagonal structure in the lepton flavor indices. This contribution will lead to a modification in the operators that are already present in the SM by substituting the implicit structure ii in the semi-leptonic terms with the more generic one ij. Moreover, NP's effect can manifest with the generation of non-negligible contributions to the chirality-flipped versions of operators whose Wilson coefficients are chirally-suppressed within the SM due to this latter's charged current $(V-A)(V-A)$ structure. Namely, the dipole and the semi-leptonic operators $\mathcal{O}_{7-10}$[7] and $\mathcal{O}_\nu$ (denoted hereafter by a primed sign) as well as scalar/pseudoscalar and tensor operators defined as [75]

$$\begin{aligned} \mathcal{O}'^{ij}_S &= \frac{e}{16\pi^2}\left(\bar{s}_{L(R)}b_{R(L)}\right)\left(\bar{l}_i l_j\right) \qquad \mathcal{O}'^{ij}_P = \frac{e}{16\pi^2}\left(\bar{s}_{L(R)}b_{R(L)}\right)\left(\bar{l}_i\gamma_5 l_j\right) \\ \mathcal{O}^{ij}_T &= \frac{e}{16\pi^2}\left(\bar{s}\sigma^{\mu\nu}b\right)\left(\bar{l}_i\sigma_{\mu\nu}l_j\right) \qquad \mathcal{O}^{ij}_{T^5} = \frac{e}{16\pi^2}\left(\bar{s}\sigma^{\mu\nu}b\right)\left(\bar{l}_i\sigma_{\mu\nu}\gamma_5 l_j\right) \end{aligned} \tag{2.18}$$

Although, due to the $SU(2)_L \otimes U(1)_Y$ invariance of the NP Lagrangian above m_{EW}, constrictions on the Wilson coefficients of both types of operators will lead to the exclusion of the tensor operators on one hand ($\mathcal{C}^{ij}_T = \mathcal{C}^{ij}_{T^5} = 0$), and to a linear dependence of the scalar/pseudoscalar operators on the other hand ($\mathcal{C}^{ij}_S = -\mathcal{C}^{ij}_P$ and $\mathcal{C}'^{ij}_S = \mathcal{C}'^{ij}_P$), which leads to a reduction in the number of free coefficients [71].
For the NC transition $B \longrightarrow K l^+ l^-$, the Lagrangian used to address $R_K^{\mu/e}$ describes NP effects in the coefficients $\mathcal{C}'_9$ and $\mathcal{C}'_{10}$. It reads

$$\mathcal{L}^{\text{NC,NP}}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\lambda^t_{bs}\left(\mathcal{C}^{ij}_9\mathcal{O}^{ij}_9 + \mathcal{C}^{ij}_{10}\mathcal{O}^{ij}_{10} + \mathcal{C}^{ij}_\nu\mathcal{O}^{ij}_\nu + \mathcal{C}'^{ij}_9\mathcal{O}'^{ij}_9 + \mathcal{C}'^{ij}_{10}\mathcal{O}'^{ij}_{10} + \mathcal{C}'^{ij}_\nu\mathcal{O}'^{ij}_\nu\right), \tag{2.19}$$

where $\mathcal{O}_9$ and $\mathcal{O}_{10}$ are defined in (2.16) (substituting lepton flavor indices (ii) with

[7] Wilson coefficients of the chirality-flipped counterparts of $\mathcal{O}_7$ and $\mathcal{O}_8$ are suppressed by m_s/m_b in $b \to s$ transitions.

(ij)), whereas, their primed versions are defined as

$$\begin{aligned}\mathcal{O}'^{ij}_9 &= \frac{e^2}{16\pi^2}\left(\bar{s}_R\gamma^\mu b_R\right)\left(\bar{l}_i\gamma_\mu l_j\right), \qquad \mathcal{O}'^{ij}_{10} = \frac{e^2}{16\pi^2}\left(\bar{s}_R\gamma^\mu b_R\right)\left(\bar{l}_i\gamma_\mu\gamma_5 l_j\right), \\ \mathcal{O}'^{ij}_\nu &= \frac{e^2}{16\pi^2}\left(\bar{s}_R\gamma^\mu b_R\right)\left(\bar{\nu}_i\gamma_\mu(1-\gamma_5)\nu_j\right).\end{aligned} \tag{2.20}$$

Thus, being negligible in the SM, primed operators are supposed to dominate in $\mathcal{L}_{\text{eff}}^{\text{NC,NP}}$, whereas, unprimed operators, already present in the SM, are modified by NP. Both the initial conditions and the anomalous dimension matrices are now known for the whole set of the Wilson coefficients $\mathcal{C}_{1-10}$ at the next-to-next-to-leading order (NNLO) in QCD and next-to-leading order (NLO) in electroweak corrections [76–79].

It should be stressed that the Lagrangian (2.19) can also be used to address the neutral-current anomalies $R_{K^*}^{\mu/e}$. In fact, since the dipole operators which should be taken into account in describing $B \longrightarrow K^* l^+ l^-$ affect $R_{K^*}^{\mu/e}$ only in the low q^2 region [80], they could be omitted in the (central) region $1.1\ GeV^2 < q^2 < 6\ GeV^2$, where the same Lagrangian (2.19) can be used for the description of both $R_K^{\mu/e}$ and $R_{K^*}^{\mu/e}$, but with primed Wilson coefficients of opposite sign since K is a pseudoscalar and K^* is a vector.

An effective approach to FCCC b-quark decays

The B-decay anomalies $R_{D^{(*)}}^{\tau/l}$ are based on the charged-current transition $B \longrightarrow D^{(*)} l\bar{\nu}$. In the SM, this decay takes place at tree level. In the presence of NP, the most general elementary charged-current Lagrangian mediating the (quark-level) semi-leptonic transitions $b \to c l_i^- \bar{\nu}_{l_j}$ reads

$$\mathcal{L}_{\text{eff}}^{\text{CC,NP}}\left(b \to c l_i^- \bar{\nu}_{l_j}\right) = -4\frac{G_F}{\sqrt{2}} V_{cb}\left(\sum_n \mathcal{C}_n^{ij}\mathcal{O}_n^{ij} + h.c.\right), \tag{2.21}$$

where the sum runs over all dimension-six operators $\mathcal{O}_n^{ij}$ (n[8] $\in \{V_{L(R)}, S_{L(R)}, T\}$) allowed by the SM gauge symmetry [81, 82]

$$\begin{aligned}
\mathcal{O}_{V_L}^{ij} &= (\bar{c}\gamma_\mu P_L b)\left(\bar{l}_i\gamma^\mu U_{ij} P_L \nu_{l_j}\right), \quad & \mathcal{O}_{V_R}^{ij} &= (\bar{c}\gamma_\mu P_R b)\left(\bar{l}_i\gamma^\mu U_{ij} P_L \nu_{l_j}\right), \\
\mathcal{O}_{S_L}^{ij} &= (\bar{c} P_L b)\left(\bar{l}_i U'_{ij} P_L \nu_{l_j}\right), & \mathcal{O}_{S_R}^{ij} &= (\bar{c} P_R b)\left(\bar{l}_i U'_{ij} P_L \nu_{l_j}\right), \\
\mathcal{O}_{T}^{ij} &= (\bar{c}\sigma^{\mu\nu} P_L b)\left(\bar{l}_i\sigma_{\mu\nu} U'_{ij} P_L \nu_{l_j}\right), &&
\end{aligned} \tag{2.22}$$

where V_{cb} is the CKM matrix element, i, j are lepton flavor indices and U stands for the PMNS matrix (U' is a mixing matrix that mixes right and left-handed massive states). The effective couplings $\mathcal{C}_{V_L}^{ij} \equiv \mathcal{C}_{V_L}^{ij}(m_b)$ are defined such that they vanish in the SM. Within this latter, the dominant operator is $\mathcal{O}_{V_L} = (\bar{c}_L\gamma_\mu b_L)(\bar{l}_L\gamma^\mu \nu_L)$ which describes the tree-level exchange of the W boson, and contributes to $\mathcal{C}_{VL}^{ij}$ with δ_{ij}. Thus, the deviation from the SM is quantified in the Wilson coefficients $\mathcal{C}_{V_{L,R}}^{ij}$, $\mathcal{C}_{S_{L,R}}^{ij}$ and $\mathcal{C}_{T}^{ij}$. The effective Lagrangian for the $b \to c l_i^- \bar{\nu}_{l_j}$ reads

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{\text{CC,NP}}\left(b \to c l_i^- \bar{\nu}_{l_j}\right) = -4\frac{G_F}{\sqrt{2}} V_{cb} \Big[&(\delta_{ij} + \mathcal{C}_{V_L}^{ij})(\bar{c}_L\gamma_\mu b_L)(\bar{l}_{iL}\gamma^\mu U_{ij}\nu_{jL}) + \mathcal{C}_{V_R}^{ij}(\bar{c}_R\gamma_\mu b_R)(\bar{l}_{iL}\gamma^\mu U_{ij}\nu_{jL}) \\
&+ \mathcal{C}_{S_L}^{ij}(\bar{c}_R b_L)(\bar{l}_{iR} U'_{ij}\nu_{jL}) + \mathcal{C}_{S_R}^{ij}(\bar{c}_L b_R)(\bar{l}_{iR} U'_{ij}\nu_{jL}) \\
&+ \mathcal{C}_{T}^{ij}(\bar{c}_R\sigma_{\mu\nu} b_L)(\bar{l}_{iR}\sigma^{\mu\nu} U'_{ij}\nu_{jL}) + h.c. \Big].
\end{aligned} \tag{2.23}$$

Once again, the SM Wilson coefficients have the same value for all lepton generations as their couplings is universal.

Favored solutions from global fit analyses

For the NC transitions, the SM contribution to the Wilson coefficients in $\mathcal{L}_{\text{eff}}^{\text{NC}}$ are known to next-to leading order (NLO) accuracy [57, 70], which are all the same for $b \longrightarrow s l^+ l^-$ transitions. In fact, their values, for the only operators which are significant in the SM, are $\mathcal{C}_9 \simeq 4.3$ and $\mathcal{C}_{10} \simeq -4.2$ at the scale $\mu = m_b$ for all lepton flavors. NP contributions, on the other hand, are investigated with the help of global fits which involve observables

[8] n indicates the vector, scalar and tensor nature of the four-fermion operator, respectively.

that are sensitive to NP's presence. An analysis based on LFU observables $R_K^{\mu/e}$ and $R_{K^*}^{\mu/e}$ on one hand and on LFU differences of $B \longrightarrow K^* l^+ l^-$ angular observables $D_{p'_{4,5}}$ on the other, shows that the fit exhibits a preference for NP in individual Wilson coefficients (1-dimensional scenario) that involve left-handed currents, namely $\mathcal{C}_9^l$ and $\mathcal{C}_{10}^l$ ($l = e, \mu$) [83]. Besides, an explanation of the measured values of both ratios in terms of primed Wilson coefficients, which correspond to the right-handed quark currents, is highly disfavored[9]. Moreover, experimental results concerning $b \longrightarrow e^+e^-$ point towards the absence of significant NP contributions to any electronic Wilson coefficient $\mathcal{C}_n^e$ [57]. Hence, the significant tensions between the SM predictions and measurements in $b \longrightarrow s$ NC processes are based on the transition $b \longrightarrow s\mu^+\mu^-$. The analyses that were destined to investigate whether these discrepancies could be softened by NP contribution, even if their choice of the observables included in the fit and the treatment of the theoretical uncertainties were different, agreed all on the fact that the tensions can be relieved by an NP effect in $\mathcal{C}_9$ that interferes destructively with the SM [64, 84–87]. In the most recent global analysis of NP in $b \longrightarrow s\mu^+\mu^-$ [87], it has been established that the tension in R_K (and R_{K^*}) can be explained by a purely vector Wilson coefficient, namely $\mathcal{C}_9 \simeq -1.01$ which is consistent with the previous global fits [64, 84–86]. Another good scenario is provided by the left-handed combination $\mathcal{C}_9 = -\mathcal{C}_{10}$. Np in pairs of Wilson coefficients (2-dimensional scenario) are also obtained in the pairs $(\mathcal{C}_9, \mathcal{C}_{10})$, $(\mathcal{C}_9, \mathcal{C}_9^{'})$ and $(\mathcal{C}_9, \mathcal{C}_{10}^{'})$ which exhibit the strongest pulls. All the pairs behave similarly when it comes to both individual Wilson coefficients: best fit points shift considerably in $\mathcal{C}_9$ whilst the other operator undergoes a small shift. Table 2.1 illustrates the best fit points and the strongest pulls for NP in one $\mathcal{C}_9$ and $\mathcal{C}_{10}$ or pairs $(\mathcal{C}_9, \mathcal{C}_{10})$, $(\mathcal{C}_9, \mathcal{C}_9^{'})$ and $(\mathcal{C}_9, \mathcal{C}_{10}^{'})$ of Wilson coefficients [86]. For the CC transitions, the SM contribution to $\mathcal{L}_{\text{eff}}^{\text{CC}}$ comes from the dominant operator that describes the tree-level exchange of the W boson which equals to 1 for all three lepton generations. NP contribution is quantified in the Wilson coefficients $\mathcal{C}_{V_{L,R}}^{ij}$, $\mathcal{C}_{S_{L,R}}^{ij}$ and $\mathcal{C}_T^{ij}$. According to several studies [82, 88, 89], the anomalies

[9] If $\mathcal{O}'_9$ and $\mathcal{O}'_{10}$ were dominant, we would have opposite anomalies since primed Wilson coefficients enter R_K and R_{K^*} with opposite signs

Wilson coefficient	best fit point	1σ	pull_{SM}
$\mathcal{C}_9$	-1.11	$[-1.28, -0.94]$	5.8σ
$\mathcal{C}_9 = -\mathcal{C}_{10}$	-0.62	$[-0.75, -0.49]$	5.3σ
$\mathcal{C}_9 = -\mathcal{C}'_9$	-1.01	$[-1.18, -0.84]$	5.4σ
$(\mathcal{C}_9, \mathcal{C}_{10})$	$(-1.01, +0.29)$		5.7σ
$(\mathcal{C}_9, \mathcal{C}'_9)$	$(-1.15, +0.41)$		5.6σ
$(\mathcal{C}_9, \mathcal{C}'_{10})$	$(-1.22, -0.22)$		5.7σ

Table 2.1: Best fit points and pulls for scenarios with NP in one (1D) or two (2D) Wilson coefficients. The 1σ best-fit ranges are shown for one-dimensional cases.

in R_{D^*}, which is exhibited in the tauonic decays $b \longrightarrow c\tau\nu$, can be accommodated by the the vector exchange. In fact, the favorable solution is the product of the two left handed currents with

$$\mathcal{C}^{ii}_{V_L} \in [0.09, 0.13], \tag{2.24}$$

whereas, scenarios of NP in both the scalar $\mathcal{C}^{ii}_{S_{L,R}}$ and the tensor $\mathcal{C}^{ii}_{T}$ effective couplings are highly disfavored [88, 90].

2.1.2 The flavor puzzle and minimal flavor violation (MFV)

As already mentioned, all possible extensions of the SM could be described by a term (2.1) in the effective Lagrangian (2.13), made up by a series of higher-dimensional operators that are invariant under the SM gauge symmetry, regardless of what the BSM scenario might be. Even though this approach has the advantage of analyzing all realistic extensions of the SM in terms of a limited number of parameters (Wilson coeffients), it makes it impossible to establish correlations of NP effects at low and high energies. In fact, from the stabilization of the mechanism of electroweak symmetry breaking, responsible for the $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$, we expect that the scale Λ, at which NP is supposed to appear, to be slightly above the electroweak scale, i.e., it cannot exceed a few TeV. Moreover, the Wilson coefficients $\mathcal{C}_n$ are expected to be strongly coupled (of $\mathcal{O}(1)$) to the effective operators that are allowed by symmetry arguments, in order for the underlying theory to be natural (no fine-tuning in the couplings). Several dimension-six operators, however, contribute to flavor changing processes with coefficients that have to be severely suppressed if the NP scale is of a few TeV (see Appendix (B)) creating thus a paradoxical situation which is referred to as the *flavor puzzle* [91] and serves as the main motivation to introduce the minimal flavor violation (MFV) hypothesis [92, 93].

The basic idea of the MFV princple is to find a symmetry argument such that $\mathcal{C}_n^{ij} = \mathcal{O}(1)$ and at the same time keep the NP scale not too far from TeV. To do so, we exploit the flavor symmetry of the SM, which is not an exact symmetry in the low-energy theory as it is broken by the Yukawa interaction, and we "promote" it to be an exact symmetry of the dynamics at the TeV scale, but also, we need to specify how it is broken in order to describe the low-energy spectrum while staying in agreement with the precise experimental tests of flavor-changing processes. In order to protect, in a consistent way, quark-flavor mixing beyond the SM, we link the flavor-violating interactions beyond the SM to the known structure of the Yukawa couplings. In a quantitative way, we assume that Y^u and Y^d are the only sources of flavor symmetry breaking also beyond the SM, i.e., we make the assumption that G_q introduced in Eq. (1.26) is an exact symmetry in the NP model and it is broken by two non-dynamical fields (*spurions*) $Y^{u,d}$ which are nothing but the promoted SM's Yukawa couplings that have non-trivial transformation properties under G_q $(SU(3)_{Q_L})$

$$Y^u \sim \left(3, \bar{3}, 1\right), \qquad Y^d \sim \left(3, 1, \bar{3}\right). \tag{2.25}$$

The role of the spurions in the breaking of the flavor symmetry would be similar to that of the Higgs in the breaking of the gauge symmetry although the MFV construction holds independently of the dynamical details of the construction. When the symmetry gets broken at high energy scales, at low energy, we would only be sensitive to the background (”*vev*”) values of the spurions, which are nothing but the ordinary Yukawa couplings $Y_{u,d}$. Thus, a theory satisfies the criterion of MFV if all higher-dimensional operators that are built from SM fields and spurions are invariant under $SU(3)_{Q_L}$ [93]. According to this criterion, we can build BSM dimension-six operators with arbitrary powers of the (dimensionless) spurions that respect the $SU(3)^5$ symmetry and its breaking

$$\left(\bar{Q}_L^i \left[Y^u (Y^u)^\dagger\right]^n_{i\neq j} \gamma_\mu Q_L^j\right)^2 \quad \approx \quad \lambda_t^{2n} V_{ti}^* V_{tj} \left(\bar{Q}_L^i \gamma_\mu Q_L^j\right)^2 . \tag{2.26}$$

Here we have moved to the mass basis (1.32) where the down-quark is diagonal, and we have made use of the smallness of all the Yukawa matrices except for the top one, and the fact that the off-diagonal elements of the CKM matrix are very suppressed[10]. As a consequence, the same type of suppression in the SM is enforced to the NP amplitude. This shows in both $\Delta F = 1$ and $\Delta F = 2$ amplitudes (Appendix (B))[11]

$$\begin{aligned}
\mathcal{A}_{\Delta F=1}(f_i \longrightarrow f_j + X) &= \mathcal{A}^{\rm SM}_{\Delta F=1} + \mathcal{A}^{\rm NP}_{\Delta F=1} \\
&= y_{f_j} V_{ti}^* V_{tj} \frac{v}{\sqrt{2}} \frac{\mathcal{C}^{ij}_{\rm SM}}{\Lambda^2_{\rm SM}} + \frac{v}{\sqrt{2}} y_{f_j} V_{ti}^* V_{tj} \frac{\mathcal{C}^{ij}_{\rm NP}}{\Lambda^2_{\rm NP}} \\
&= \mathcal{A}^{\rm SM}_{\Delta F=1} \left[1 + a_1 \frac{\Lambda^2_{\rm SM}}{\Lambda^2_{\rm NP}}\right],
\end{aligned} \tag{2.27}$$

and

$$\begin{aligned}
\mathcal{A}_{\Delta F=2}(M_i \longrightarrow M_j) &= \mathcal{A}^{\rm SM}_{\Delta F=2} + \mathcal{A}^{\rm NP}_{\Delta F=2} \\
&= y_t^2 (V_{ti}^* V_{tj})^2 \frac{\mathcal{C}^{ij}_{\rm SM}}{\Lambda^2_{\rm SM}} + y_t^2 (V_{ti}^* V_{tj})^2 \frac{\mathcal{C}^{ij}_{\rm NP}}{\Lambda^2_{\rm NP}} \\
&= \mathcal{A}^{\rm SM}_{\Delta F=2} \left[1 + a_2 \frac{\Lambda^2_{\rm SM}}{\Lambda^2_{\rm NP}}\right],
\end{aligned} \tag{2.28}$$

where $y_f = \sqrt{2} m_f / v$, $\Lambda_{\rm SM} = 4\pi v \approx 3$ TeV and $a_{1,2} = \mathcal{C}^{ij}_{\rm NP}/\mathcal{C}^{ij}_{\rm SM} \sim \mathcal{O}(1)$. Thus, with the experimental condition that $\left|\mathcal{A}^{\rm NP}\right| < \left|\mathcal{A}^{\rm SM}\right|$, the bound on the NP scale would be within the reach of LHC, even with $\mathcal{C}_{\rm NP} \sim 1$ which would correspond to a strongly coupled NP sector.

The (real) parameters a_i do not depend on the flavor; they depend only on the con-

[10] $\left[Y^u (Y^u)^\dagger\right]^n_{i\neq j} = y_u^{2n} V_{ui}^* V_{uj} + y_c^{2n} V_{ci}^* V_{cj} + y_t^{2n} V_{ti}^* V_{tj}$ and $y_{u,c,t} = \frac{\sqrt{2}}{v} m_{u,c,t}$.

[11] For $\Delta F = 1$ operator, the additional factor of spurions is $Y^d (Y^u)^\dagger Y^u$.

sidered operator, meaning that flavor-changing transitions of the same type such as $b \longrightarrow s\gamma$ and $s \longrightarrow d\gamma$, which have the same structure of the dimension-six operator $(eF_{\mu\nu}H\bar{Q}_L^i\sigma^{\mu\nu}d_R^j)$, would have the exact same deviation. Thus, a positive evidence of NP exhibiting the flavor-universality pattern in transitions of the same type, would be a valid proof of the MFV hypothesis. While the validity of this hypothesis is still far from being proved from data, the MFV framework, not only seems to be a natural solution to the flavor problem, but also serves as a predictive approach in flavor physics. Moreover, it can demonstrate, in case falsified, that not only there is physics beyond the SM, but also it would be a clear signal of new sources of flavor symmetry breaking beyond (in addition to) the Yukawa couplings.

2.2 Model dependant approach

As the anomalies fit into the coherent patterns of lepton flavor universality violation beyond the SM, it would be natural to call for an extension of this latter where new particles and new interactions are involved. In fact, many scenarios that involve either scalar/vector bosons and fermions have been proposed. Most of the successful candidates can be cast into two sets:

- **Leptoquarks (LQs)**, which are hypothetical particles that carry color and can turn quarks into leptons and vice-versa. They were first proposed in the context of the Pati-Salam model [95] and the Grand Unified Theories (GUTs) [96, 97]. LQs can be either $SU(2)_L$ scalars or vectors in the singlet and triplet representations of the $SU(2)_L$ gauge group. Among the various scenarios of scalar leptoquarks that were proposed [98], the most favored ones are the weak singlet scalar LQ with hypercharge 1/3, namely $S_1 \sim (\bar{3}, 1)_{1/3}$, and the triplet of scalar LQ states with hypercharge 1/3, $S_3 \sim (\bar{3}, 3)_{1/3}$, where the LQs are specified by their SM quantum numbers $(SU(3)_C, SU(2)_L)_Y$ with the electric charge Q being the sum of the weak hypercharge Y and the third weak isospin component I_3. Their relevant interaction Lagrangians read respectively

$$\begin{aligned}\mathcal{L}_{S_1} &= Y_L^{ij}\overline{Q_i^{cL}}i\tau_2 L_j^L S_1 + Y_R^{ij}\overline{u_i^{cR}}l_j^R S_1 + h.c.,\\ \mathcal{L}_{S_3} &= Y_L^{ij}\overline{Q_i^{cL}}i\tau_2 L_j^L(\tau_k S_3^k) + h.c.,\end{aligned} \tag{2.29}$$

where $Y_{L,R}$ are the Yukawa matrices, τ are the Pauli matrices and S_3^k is the k component of the LQ triplet. Among the vector LQs that were proposed, the weak singlet leptoquark $U_1 \sim (\bar{3}, 1)_{2/3}$ appears to stand out as an excellent candidate as it can provide a simultaneous explanation for both anomalies [24, 27]. The weak triplet LQ $U_3 \sim (3, 3)_{2/3}$ also can accommodate the anomaly $R_{K^{(*)}}$ in a similar way to that of the U_1 model [99]. Their most general Lagrangians consistent with the SM gauge symmetry read respectively

$$\begin{aligned}\mathcal{L}_{U_1} &= g_L^{ij}\overline{Q_i^L}\gamma_\mu U_1^\mu L_j^L + g_R^{ij}\overline{d_i^R}\gamma_\mu U_1^\mu l_j^R + h.c.,\\ \mathcal{L}_{U_3} &= g_L^{ij}\overline{Q_i^L}\gamma_\mu(\tau_k U_3^{k\mu})L_j^L + h.c.,\end{aligned} \tag{2.30}$$

where $g_{L,R}^{ij}$ are the couplings and U_3^k is the k component of the U_3 LQ in $SU(2)_L$ space.

- **Color-less vectors**, which can either be electrically neutral or singly charged (QCD neutral) heavy spin-1 particles. The effective operators are obtained by integrating out heavy color-less $SU(2)_L$ singlet $Z'_\mu \sim (1, 1)_0$ and triplet $W'_\mu \sim (1, 3)_0$ coupled weakly to the SM fermion singlet and triplet currents, respectively [17]. Within this scenario, the process is mediated by heavy exotic gauge bosons whose couplings depend on the generation.

In order to cope with the electroweak's and the purely leptonic observables' constraints, some degree of model-building effort is required. For this purpose, a model of a particular interest, namely the 331 model, that embeds heavy gauge bosons (W', Z') is proposed as a possible BSM scenario that could accommodate the B anomalies in both the NC and the CC sectors.

Chapter 3

Lepton flavor universality violation in the 331 model

Among the many scenarios that have been proposed to extend the SM is one that is based on the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [100–102]. This set constitutes one of the simplest extensions of the SM, where this latter is recovered after a first spontaneous symmetry breakdown that occurs at a heavier Λ_{NP}. Subsequently, the SM gauge group gets broken down spontaneously in its turn to $U(1)_{em}$ at the lower scale Λ_{EW}.

In 331 models, the SM gauge group is extended to the broader $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ with $SU(3)_L$ generators being $\hat{T}^a = \frac{\lambda^a}{2}$ (λ^a are the Gell-Mann matrices, with $a = 1, .., 8$) and $\hat{T}^9$, defined as $\hat{T}^9 = \frac{\mathbb{1}}{\sqrt{6}}$[1] generates $U(1)_X$ ($\mathbb{1}$ is the 3×3 unit matrix). The hypercharge operator $\hat{Y}$ is defined in terms of the generators of the new gauge group by requiring that it commutes with all its generators, i.e., it would have only terms proportional to $\hat{T}^8$ and $U(1)_X$'s generator X

$$\frac{\hat{Y}}{2} = \beta \hat{T}^8 + X\mathbb{1}, \tag{3.1}$$

[1]The $U(1)_X$ generator satisfies the same normalization relation as the eight generators of $SU(3)_L$: $\text{Tr}[\hat{T}^a \hat{T}^b] = \frac{\delta^{ab}}{2}$.

with β is a parameter that distinguishes different 331 models, and

$$\hat{T}^8 = \frac{1}{2}\hat{\lambda}^8 = \frac{1}{2\sqrt{3}}\ \mathrm{diag}(1,1,-2). \tag{3.2}$$

In analogy with the SM, we introduce the electric charge operator defined as a linear combination of the diagonal generators of the group

$$\hat{Q} = \hat{T}^3 + \frac{\hat{Y}}{2}, \tag{3.3}$$

with $\hat{T}^3 = \frac{1}{2}\hat{\lambda}^3 = \frac{1}{2}\ \mathrm{diag}(1,-1,0)$. The parameter β, encodes the way in which the $SU(3)_L \otimes U(1)_X$ is embedded in $SU(2)_L \otimes U(1)_Y$ as it controls the relation between the hypercharge and the generator $\hat{T}^8$ of $SU(3)_L$. From Eqs. (3.1) and (3.3), its value reads

$$\beta = \mp\frac{1}{\sqrt{3}}\left(2Q' - \frac{1}{3}\right), \tag{3.4}$$

where Q' is the electric charge of the *exotic* (third additional massive) quark[2]. Thus, by choosing to identify the first two components of the $SU(3)_L$ triplet (or antitriplet) with the SM doublet, and by demanding that no new quark introduced in the model has exotic charges, value of β is restricted to $\pm\frac{1}{\sqrt{3}}$[3]. The value of β plays a key role in distinguishing various versions of the model and only for some of its values, the gauge bosons turn out to have integer charges.

Requirement of quantum-anomalies cancellation puts stringent constrains for 331 model building. For a theory to be anomaly-free, several relations among the fermion charges have to be satisfied [103–105], namely

$$[SU(3)_C]^2 \otimes U(1)_X \quad \Rightarrow \quad 3\sum_Q X_Q^L = \sum_q X_q^R, \tag{3.5}$$

[2]In the of the leptons, $\beta = \mp\frac{1}{\sqrt{3}}\,(2Q'+1)$, with Q' being the electric charge of the exotic lepton.

[3]The value $\pm\sqrt{3}$, commonly chosen in the literature, introduces exotic electric charges for the third entry of the quark triplet (or antitriplet) 5/3 and $-4/3$.

$$[SU(3)_L]^3 \text{ anomaly} \quad \Rightarrow \quad N_3 = N_{\bar{3}}, \tag{3.6}$$

$$[SU(3)_L]^2 \otimes U(1)_X \quad \Rightarrow \quad 3\sum_Q X_Q^L + \sum_l X_l^L = 0, \tag{3.7}$$

$$[\text{Grav}]^2 \otimes U(1)_X \quad \Rightarrow \quad 9\sum_Q X_Q^L + 3\sum_l X_l^L = 3\sum_q X_q^R + \sum_s X_s^R, \tag{3.8}$$

$$[U(1)_X]^3 \quad \Rightarrow \quad 9\sum_Q (X_Q^L)^3 + 3\sum_l (X_l^L)^3 = 3\sum_q (X_q^R)^3 + \sum_s (X_s^R)^3, \tag{3.9}$$

where Q and q denote the quark left-handed generations and the corresponding singlets, respectively. L denotes the lepton multiplets and s the corresponding singlets. In order to cancel the $[SU(3)_L]^3$ anomaly, equation (3.6) states that the number of triplets and antitriplets has to be equal. However, LFUV couplings for the gauge bosons cannot be generated unless we introduce additional lepton generations. In fact, if we call N_Q, N_L ($N_{\bar{Q}}$, $N_{\bar{L}}$) the number of quark, lepton generations transforming as 3 ($\bar{3}$), respectively, (3.6) yields

$$3N_Q + N_L = 3N_{\bar{Q}} + N_{\bar{L}}. \tag{3.10}$$

Several possibilities arise. Assuming that all three quark families transform in the same way would lead to at least nine lepton generations all transforming in the opposite way. This would not lead to the generation of different couplings between the leptons and the gauge bosons. Hence, LFUV arises only when quark families transform differently from one another. Assuming that two quark families transform as $\bar{3}$, Eq. (3.10) yields

$$N_L - N_{\bar{L}} = 3. \tag{3.11}$$

If we assume that the lepton generation number is three, then they would all transform in the same way. In this minimal construction [106], usually considered in the literature, there would be no LFUV couplings generated from identical couplings of the gauge bosons to all lepton generations as they transform identically. Then, additional lepton generations have to be embedded in the theory. The minimal possibility to solve equation (3.11) is to increase the number of lepton families by two, so that $N_L = 4$ and

$N_{\bar{L}} = 1$ generates LFUV in the gauge couplings to leptons.

It is worth to stress that neither the minimal construction, which is based on placing the left-handed lepton doublets in $SU(3)_L$ triplets, nor the flipped set-up [107], which is based on perfect quark family replication, generates LFUV from couplings of the gauge bosons to the fermions. In fact, in both scenarios, the gauge bosons couple identically to all lepton and quark families, respectively. In order to generate LFUV from different couplings of the gauge bosons to all fermionic fields, we adopt an anomaly-free non-minimal 331 set where the leptons are grouped in no less than five generations.

3.1 The $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with $\beta = 1/\sqrt{3}$

In order to generate potential LFUV effects, we adopt a non-minimal construction that ensures the cancellation of the anomalies and allows for different representations for the lepton generations. By demanding that no introduced degree of freedom has exotic electric charge, we choose $\beta = \frac{1}{\sqrt{3}}$.

3.1.1 Fields and representations

In what follows, the SM fermions are labeled with lower cases and the exotic ones with upper cases. We introduce the left-handed components of the fields together with the right-handed partners of the charged ones where the representations of the particles are referred to with the notation $(SU(3)_C, SU(3)_L, U(1)_X)$[4]

(i) three generations of quarks

$$Q_m^L = \begin{pmatrix} d_m \\ -u_m \\ U_m \end{pmatrix}; \qquad u_m^R;\ d_m^R;\ U_m^R, \qquad (m = 1, 2)\,, \tag{3.12}$$

[4]The fields in the antitriplet are fixed with the minus sign in order to reproduce the SM couplings.

with $(SU(3)_C, SU(3)_L, U(1)_X) = \left(3, \bar{3}, \frac{1}{3}\right), (3, 1, 2/3), (3, 1, -1/3), (3, 1, 2/3)$.

$$Q_3^L = \begin{pmatrix} u_3 \\ d_3 \\ D_3 \end{pmatrix}; \qquad u_3^R;\ d_3^R;\ D_3^R, \tag{3.13}$$

with $(SU(3)_C, SU(3)_L, U(1)_X) = (3, 3, 0), (3, 1, 2/3), (3, 1, -1/3), (3, 1, -1/3)$.

(ii) five generations of leptons

$$l_1^L = \begin{pmatrix} e_1^{-L} \\ -\nu_1^L \\ N_1^L \end{pmatrix}; \qquad e_1^{-R}, \tag{3.14}$$

with $(SU(3)_C, SU(3)_L, U(1)_X) = \left(1, \bar{3}, -1/3\right), (1, 1, -1)$.

$$l_n^L = \begin{pmatrix} \nu_n^L \\ e_n^{-L} \\ E_n^{-L} \end{pmatrix}; \qquad e_n^{-R};\ E_n^{-R}, \qquad (n = 2, 3), \tag{3.15}$$

with $SU(3)_C \times SU(3)_L \times U(1)_X$ quantum numbers $(1, 3, -2/3), (1, 1, -1), (1, 1, -1)$.

$$l_4^L = \begin{pmatrix} N_4^L \\ E_4^{-L} \\ F_4^{-L} \end{pmatrix}; \qquad F_4^{-R}, \tag{3.16}$$

with $(SU(3)_C, SU(3)_L, U(1)_X) = (1, 3, -2/3), (1, 1, -1)$.

$$l_5^L = \begin{pmatrix} \left(E_4^{-R}\right)^c \\ N_5^L \\ P_5^L \end{pmatrix}, \tag{3.17}$$

with $(SU(3)_C, SU(3)_L, U(1)_X) = (1, 3, 1/3)$.

This construction contains no positively charged leptons. They would have appeared, however, in l_5^L had we considered the original set (Model B in Ref. [104]). In fact, originally the model contained fifteen leptons instead of fourteen (eight charged and seven neutral ones) with one positively charged lepton belonging to the l_5^L triplet that had to be identified with the charge conjugate of the right handed partner of the fourth generation. This procedure ensures the removal of an unwanted electroweak-scale mass term of a charged exotic particle that would appear after the symmetry gets broken down spontaneously (see Section (3.1.2)). In what follows, it proves easier to discuss the spectrum of the theory after the introduction of the flavor vectors where the fields of the same electric charge are gathered

$$\begin{aligned}
&U = (u_1, u_2, u_3, U_1, U_2)^T\,,\\
&D = (d_1, d_2, d_3, D_3)^T\,,\\
&f_L^- = \left(e_1^{-L}, e_2^{-L}, e_3^{-L}, E_2^{-L}, E_3^{-L}, E_4^{-L}, F_4^{-L}\right)^T,\\
&f_R^- = \left(e_1^{-R}, e_2^{-R}, e_3^{-R}, E_2^{-R}, E_3^{-R}, E_4^{-R}, F_4^{-R}\right)^T,\\
&N_L = \left(\nu_1^L, \nu_2^L, \nu_3^L, N_1^L, N_4^L, N_5^L, P_5^L\right)^T.
\end{aligned} \tag{3.18}$$

The right handed partners for the neutral particles, which would be pure singlets with respect to the whole gauge group, are left out of the discussion as they are of no relevance to our analysis. They would be of improtance, though, when we discuss neutrino masses (see Section (3.1.3)).

3.1.2 Scalar sector and particles' spectrum

The 331 models experience two stages of SSBs: one occuring at the heavier scale $\Lambda_{\rm NP}$, after which the SM is recovered and all exotic charged particles acquire mass, and another one that occurs at the lower energy scale $\Lambda_{\rm EW}$. These models feature an extended Higgs sector that triggers the two symmetry breakdowns leading to heavy exotic new degrees of freedom as well as the usual SM's.

The scalar sector

The model undergoes two stages of spontaneous symmetry breakings (SSB). The first step leads to recovering the SM as the low-energy effective theory deriving from the 331 model, while the subsequent step is the familiar SM SSB realized at the electroweak scale. The overal pattern of SSB follows the scheme
$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{\langle\Phi_1\rangle} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\langle\Phi_2\rangle} SU(3)_C \otimes U(1)_Q$,
where Φ_1 and Φ_2 are the two scalars whose *vevs* trigger the two stages of SSB. In the first breakdown, five gauge fields acquire mass of the order of $\langle\Phi_1\rangle$, and in the second (electroweak) breakedown, three gauge fields acquire mass proportional to $\langle\Phi_2\rangle$. The remaining gauge field which is associated with the unbroken generator Q remains masseless and is identified with the photon. In the first transition $SU(3)_L \otimes U(1)_X \xrightarrow{\langle\Phi_1\rangle} SU(2)_L \otimes U(1)_Y$, the *vev* of Φ_1 should accomplish the following conditions

$$\left[\hat{T}_L^{1,2,3}, \langle\Phi_1\rangle\right] = \left[\hat{Q}, \langle\Phi_1\rangle\right] = 0, \tag{3.19}$$

which means that five gauge bososns acquire mass of the order of $\langle\Phi_1\rangle$, while the other generators give a non-vanishing result when acting on the vacuum. The second transition $SU(2)_L \otimes U(1)_Y \xrightarrow{\langle\Phi_2\rangle} U(1)_Q$, triggered by the *vev* of Φ_2, the conditions that should be satisfied read

$$\left[\hat{T}_L^{1,2,3}, \langle\Phi_2\rangle\right] \neq 0, \qquad \left[\hat{Q}, \langle\Phi_2\rangle\right] = 0, \tag{3.20}$$

where three gauge bosons gain mass and the only generator that leaves the vacuum invariant should be Q. Since the goal is to generate masses to the fermions and the gauge bosons (except for the photon), representations for the scalar fields are constrained from the Yukawa terms i.e. couplings between a scalar field and two fermions, in order to obtain appropriate mass terms after the SSB. Since the fermions transform either as 3 or as a $\bar{3}$ under $SU(3)_L$, possible representations of the ($SU(3)_C$ singlet) scalar fields are built from gauge invariant terms for the fermion-fermion-scalar couplings

$$\begin{aligned}
&\bar{\psi}_L^i \psi_R \Phi: \quad \bar{3} \otimes 1 \otimes \Phi = 1 \quad \Rightarrow \quad \Phi = 3, \\
&\bar{\psi}_L^i (\psi_L)^c \Phi: \quad \bar{3} \otimes \bar{3} \otimes \Phi = 1 \quad \Rightarrow \quad \Phi = \bar{3} \oplus 6, \\
&\bar{\psi}_R (\psi_R)^c \Phi: \quad 1 \otimes 1 \otimes \Phi = 1 \quad \Rightarrow \quad \Phi = 1, \\
&\bar{\psi}_R (\psi_L^i)^c \Phi: \quad 1 \otimes \bar{3} \otimes \Phi = 1 \quad \Rightarrow \quad \Phi = 3.
\end{aligned} \tag{3.21}$$

Thus, in order to generate masses for the fermions, the scalar field can only be a singlet, a triplet or a sextet, although, after the two stages of SSB, the *vev* of the singlet scalar will never give rise to a mass term for the gauge bosons or the fermions, because the electromagnetic invariance makes it a scalar under $U(1)_X$[5]. Thus, we will omit this possibility. For the first transition 331 $\rightarrow$ 321, we denote with χ (χ^*) and S_a the triplet (antitriplet) and sextet, respectively, whose *vev*s and $U(1)_X$ charges are aligned according to the conditions (3.19) that ensure the non-breaking of $SU(2)_L$ nor $U(1)_Q$ at the scale identified with $\langle \Phi_1 \rangle$ ($\Phi_1 \in \{\chi, \chi^*, S_a\}$)[6], and impose a requirement on the quantum numbers of the representations. Their *vev*s align in the following way [108]

$$\langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \sim (1, 3, \frac{1}{3}), \qquad \langle S_a \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{pmatrix} \sim (1, 6, \frac{2}{3}). \tag{3.22}$$

The subsequent SSB 321 $\rightarrow$ 31 occurs at the electroweak energy scale. It reproduces the EWSB of the SM and is accomplished by means of two triplets denoted η and ρ and three sextets $S_{b,c,d}$ ($\Phi_2 \in \{\eta, \rho, S_b, S_c, S_d\}$). Their *vev*s and $U(1)_X$ charges are aligned according to the condition (3.20). The electric charge of the components of the scalar

[5] A singlet always commutes with the electric charge generator Q.

[6] The action of the $SU(3)$ matrices T^a on a sextet S is defined as: $\hat{T}^a S = S T^a + (T^a)^T S$.

fields of the representations 3, $\bar{3}$, and 6 are respectively given by [108]

$$Q_3 = \begin{pmatrix} \frac{2}{3} + X_{\Phi_2} \\ -\frac{1}{3} + X_{\Phi_2} \\ -\frac{1}{3} + X_{\Phi_2} \end{pmatrix}, \qquad Q_{\bar{3}} = \begin{pmatrix} -\frac{2}{3} - X_{\Phi_2^*} \\ \frac{1}{3} - X_{\Phi_2^*} \\ \frac{1}{3} - X_{\Phi_2^*} \end{pmatrix},$$
$$Q_6 = \begin{pmatrix} \frac{4}{3} + X_{\Phi_2} & \frac{1}{3} + X_{\Phi_2} & \frac{1}{3} + X_{\Phi_2} \\ \frac{1}{3} + X_{\Phi_2} & -\frac{2}{3} + X_{\Phi_2} & -\frac{2}{3} + X_{\Phi_2} \\ \frac{1}{3} + X_{\Phi_2} & -\frac{2}{3} + X_{\Phi_2} & -\frac{2}{3} + X_{\Phi_2} \end{pmatrix}. \tag{3.23}$$

The right alignment is chosen by imposing a zero charge to each representation and verify if $U(1)_X$-invariant Yukawa terms involving these scalar fields can be built. The *vev*s responsible for the EWSB are thus aligned as follows

$$\langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w_2 \\ w_3 \end{pmatrix} \sim (1, 3, \frac{1}{3}), \qquad \langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \sim (1, 3, -\frac{2}{3}),$$
$$\langle S_b \rangle = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim (1, 6, -\frac{4}{3}), \qquad \langle S_c \rangle = \begin{pmatrix} 0 & c_1 & c_2 \\ c_1 & 0 & 0 \\ c_2 & 0 & 0 \end{pmatrix} \sim (1, 6, -\frac{1}{3}), \tag{3.24}$$
$$\langle S_d \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & d_1 & d_2 \\ 0 & d_2 & d_3 \end{pmatrix} \sim (1, 6, \frac{2}{3}).$$

Gauge bosons' spectrum

Despite the fact that the fermion content may vary significantly from one construction to another, all $SU(3)_L \otimes U(1)_X$ models have the same gauge sector. The eight gauge bosons of the $SU(3)_L$ group, which transform according to its adjoint representation, are denoted as W_μ^a, $a = 1, ..8$, while the gauge boson of the $U(1)_X$ group is denoted by X_μ. Denoting by g the coupling constant of the fermions to the gauge bosons of the $SU(3)_L$ group and by g_X the coupling constants of the fermions to X_μ, and by X the quantum number corresponding to $U(1)_X$, the covariant derivatives acting on the triplets, antitriplets and singlets, respectively, read

$$\begin{aligned} D_\mu \psi_L &= \partial_\mu \psi_L - igW^a_\mu \hat{T}^a \psi_L - ig_X X X_\mu \hat{T}^9 \psi_L, \\ D_\mu \bar{\psi}_L &= \partial_\mu \bar{\psi}_L + igW^a_\mu (\hat{T}^a)^T \bar{\psi}_L - ig_X X X_\mu \hat{T}^9 \bar{\psi}_L, \\ D_\mu \psi_R &= \partial_\mu \psi_R - ig_X X X_\mu \hat{T}^9 \psi_R, \end{aligned} \tag{3.25}$$

where the generators in the case of antitriplets are $\hat{\bar{T}}_a = -(\hat{T}^a)^T$[7]. The matrix $W^a_\mu \hat{T}^a$ is arranged as follows

$$W_\mu = W^a_\mu \hat{T}^a = \frac{1}{2} \begin{pmatrix} W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} W^+_\mu & \sqrt{2} Y^{Q_Y}_\mu \\ \sqrt{2} W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} V^{Q_V}_\mu \\ \sqrt{2} Y^{-Q_Y}_\mu & \sqrt{2} V^{-Q_V}_\mu & -\frac{2}{\sqrt{3}} W^8_\mu \end{pmatrix}, \tag{3.26}$$

where the following definitions have been introduced

$$\begin{aligned} W^\pm_\mu &= \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu), \\ Y^{\pm Q_Y}_\mu &= \frac{1}{\sqrt{2}} (W^4_\mu \mp i W^5_\mu), \\ V^{\pm Q_V}_\mu &= \frac{1}{\sqrt{2}} (W^6_\mu \mp i W^7_\mu). \end{aligned} \tag{3.27}$$

As mentioned earlier, the value of the parameter β plays a key role in the model as it controls, not only the charges of the new (exotic) fermions, but also the electric charges of the heavy gauge bosons. In fact, from the action[8] of the electric charge operator (3.3) on the gauge bosons matrix $\hat{Q}_W W_\mu = [Q, W_\mu] = Q^{GB} W_\mu$ we can read the charges

[7] T is for transpose.

[8] The action of operators should be distinguished from the simple multiplication with the corresponding matrix.

of the entries in Eq. (3.26) [109]

$$Q^{GB} = \begin{pmatrix} 0 & 0 & \frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -1 & 0 & -\frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -\frac{1}{2} - \beta\frac{\sqrt{3}}{2} & \frac{1}{2} - \beta\frac{\sqrt{3}}{2} & 0 \end{pmatrix}. \tag{3.28}$$

It is clear that only for some values of β, the gauge bosons have integer electric charges. For our specific choice of $\beta = \frac{1}{\sqrt{3}}$, the matrix $W_\mu^a \hat{T}^a$ is arranged as follows

$$W_\mu = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} W_\mu^+ & \sqrt{2} Y_\mu^+ \\ \sqrt{2} W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} V_\mu^{+0} \\ \sqrt{2} Y_\mu^- & \sqrt{2} V_\mu^{-0} & -\frac{2}{\sqrt{3}} W_\mu^8 \end{pmatrix}, \tag{3.29}$$

where the spectrum exhibits the two (SM's) charged and three neutral gauge bosons, plus other four exotic ones, which depending of the chosen value of β[9], turn out to be two singly charged $Y^\pm$ and two additional neutral bosons V^0 and V^{-0}.

At the first SSB, five gauge fields acquire mass of the order of Λ_{NP}, whereas, three of the remaining four gauge bosons will become massive at the electroweak scale. These stem from the covariant derivative in the Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} \propto (D_\mu \chi)^\dagger (D^\mu \chi) + (D_\mu \eta)^\dagger (D^\mu \eta) + (D_\mu \rho)^\dagger (D^\mu \rho) + \sum_{i=a,b,c,d} Tr\left\{ (D_\mu S_i)^\dagger (D^\mu S_i) \right\}. \tag{3.30}$$

[9] For comparison, for $\beta = \sqrt{3}$, the four additional bosons are two singly charged bosons $V^\pm$ and two doubly charged ones Y^{++} and Y^{--}.

The masses of the $W^{\pm}$, $Y_\mu^{\pm}$ and V_μ^0 read

$$\begin{aligned}
M_{W^\pm}^2 &= \frac{g^2}{4}\left[v^2 + w_2^2 + 2\left(b^2 + 2c_1^2 + c_2^2 + d_1^2 + d_2^2\right)\right], \\
M_{Y^\pm}^2 &= \frac{g^2}{4}\left[\left(u^2 + 2a^2\right) + v^2 + w_3^2 + 2\left(b^2 + c_1^2 + 2c_2^2 + d_2^2 + d_3^2\right)\right], \\
M_{V^0}^2 &= \frac{g^2}{4}\left[\left(u^2 + 2a^2\right) + w_2^2 + w_3^2 + 2\left(d_1^2 + 2d_2^2 + d_3^2\right)\right].
\end{aligned} \tag{3.31}$$

In a first step, four gauge bosons W_μ^4, W_μ^5 $(Y_\mu^{\pm})$, W_μ^6 and W_μ^7 (V_μ^0) become massive while the two neutral gauge bosons W_μ^8 and X_μ mix together giving rise to a massive Z'_μ and a massesless one B_μ. The mixing angle is denoted by θ_{331}

$$\begin{pmatrix} Z'_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{331} & \sin\theta_{331} \\ -\sin\theta_{331} & \cos\theta_{331} \end{pmatrix} \begin{pmatrix} X_\mu \\ W_\mu^8 \end{pmatrix}, \tag{3.32}$$

where

$$\sin\theta_{331} = \frac{g}{\sqrt{g^2 + \frac{g_X^2}{18}}} \qquad \text{and} \qquad \cos\theta_{331} = \frac{\frac{g_X}{3\sqrt{2}}}{\sqrt{g^2 + \frac{g_X^2}{18}}}. \tag{3.33}$$

Here, g and g_X are the gauge coupling constants.

After the EWSB, the neural bosons W_μ^3 and B_μ mix together with a mixing angle $\theta_{\mathcal{W}}$ (Weinberg angle) to yield a massless A_μ identified with the photon, and a massive Z_μ

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathcal{W}} & -\sin\theta_{\mathcal{W}} \\ \sin\theta_{\mathcal{W}} & \cos\theta_{\mathcal{W}} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{3.34}$$

with

$$M_Z^2 = \frac{g^2}{4\cos\theta_{\mathcal{W}}}\left[v^2 + w_2^2 + 2\left(b^2 + 2c_1^2 + c_2^2 + d_1^2 + d_2^2\right)\right]. \tag{3.35}$$

The two mixing angles θ_{331} and $\theta_{\mathcal{W}}$ and the two gauge coupling constants obey the relations

$$\cos\theta_{331} = \frac{1}{\sqrt{3}}\tan\theta_{\mathcal{W}} \qquad \text{and} \qquad \frac{g}{g_X} = \frac{\tan\theta_{331}}{3\sqrt{2}}. \tag{3.36}$$

From Eqs. (3.35) and (3.31), we get, as in the SM, $M_W^2/M_Z^2 = \cos\theta_{\mathcal{W}}$, which justifies the identification of the angle $\theta_{\mathcal{W}}$ with the Weinberg angle.

The Yukawa sector

After the two stages of spontaneous symmetry breaking, all (charged) fermion fields of the model get mass from their coupling to the Higgs mutliplets. The SSB pattern, though, should generate heavy masses to the exotic fields only after the first SSB which occurs at the high energy scale $\Lambda_{\rm NP}$, leaving only three masseless fermion fields whose mass generation should be due to the second electroweak braking.

- **Quark mass terms**

  The Yukawa Lagrangian containing all gauge invariant quark-quark-scalar terms responsible for the quark masses is

$$\mathcal{L}_Y^q = \left(\bar{Q}_L^m \chi^* Y_{mi}^u + \bar{Q}_L^m \eta^* j_{mi}^u + \bar{Q}_L^3 \rho y_{3i}^u\right) U_i^R + \left(\bar{Q}_L^3 \chi Y_{3j}^d + \bar{Q}_L^3 \eta j_{3j}^d + \bar{Q}_L^m \rho^* y_{mj}^d\right) D_j^R, \tag{3.37}$$

  where

  $U_i^R = \left(u_1^R, u_2^R, u_3^R, U_1^R, U_2^R\right)^T.$

  $D_j^R = \left(d_1^R, d_2^R, d_3^R, D_3^R\right)^T.$

  $Y^{u,d}$, $y^{u,d}$ and $j^{u,d}$ are the Yukawa couplings for χ, ρ and η, respectively.

  The mass matrix of the quarks can be written in terms of the flavor vectors introduced in Eq. (3.18) as

$$M_q = \bar{U}_L M_u U_R + \bar{D}_L M_d D_R, \tag{3.38}$$

where

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} j^u_{11} w_1 & j^u_{12} w_1 & j^u_{13} w_1 & j^u_{14} w_1 & j^u_{15} w_1 \\ j^u_{21} w_1 & j^u_{22} w_1 & j^u_{23} w_1 & j^u_{24} w_1 & j^u_{25} w_1 \\ y^u_{31} v & y^u_{32} v & y^u_{33} v & y^u_{34} v & y^u_{35} v \\ Y^u_{11} u + j^u_{11} w_2 & Y^u_{12} u + j^u_{12} w_2 & Y^u_{13} u + j^u_{13} w_2 & Y^u_{14} u + j^u_{14} w_2 & Y^u_{15} u + j^u_{15} w_2 \\ Y^u_{21} u + j^u_{21} w_2 & Y^u_{22} u + j^u_{22} w_2 & Y^u_{23} u + j^u_{23} w_2 & Y^u_{24} u + j^u_{24} w_2 & Y^u_{25} u + j^u_{25} w_2 \end{pmatrix}, \tag{3.39}$$

and

$$M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y^d_{11} v & y^d_{12} v & y^d_{13} v & y^d_{14} v \\ y^d_{21} v & y^d_{22} v & y^d_{23} v & y^d_{24} v \\ j^d_{31} w_2 & j^d_{32} w_2 & j^d_{33} w_2 & j^u_{34} w_2 \\ Y^d_{31} u + j^d_{31} w_3 & Y^d_{32} u + j^d_{32} w_3 & Y^d_{33} u + j^d_{33} w_3 & Y^d_{34} u + j^d_{34} w_3 \end{pmatrix}. \tag{3.40}$$

The diagonalization of both mass matrices before the EWSB (in the limit $v = w_1 = w_2 = 0$) leaves three masseless quarks for both the up and down components, meaning that after the $SU(3)_L$ SSB, all the new exotic quarks acquire mass of the Λ_{NP} scale, and that the three remaining masseless (SM) quarks should get mass at the electroweak scale. This is why such exotic particles have not yet been observed at the electroweak scale.

- **Charged lepton mass terms**

  The model considered with $\beta = 1/\sqrt{3}$ consisted originally of five exotic charged leptons instead of four defined in Eqs. (3.15), (3.16) and (3.17), where one positively charged lepton belongs to the fifth generation

$$l^L_5 = \begin{pmatrix} E^{+L}_5 \\ P^L_5 \\ N^L_5 \end{pmatrix} \sim (1, 3, 1/3)\,, \qquad E^{+R}_5 \sim (1, 1, 1)\,.$$

After the first SSB, the Yukawa Lagrangian containing all gauge invariant lepton-

lepton-scalar terms for the negatively charged leptons reads

$$\mathcal{L}_Y^{l^-} \supset Y\chi^* \bar{l}_n^L (l_5^L)^c + Y\chi \bar{l}_n^L \left(e_m^{-R} + E_n^{-R} + E_5^{-R}\right), \tag{3.41}$$

where Y is the Yukawa coupling of the left-handed lepton fields l_n with the scalar χ[10], $n = 2, 3, 4$ and $m = 1, 2$. The combination of $SU(3)_L$ triplets and antitriplets is

$$\epsilon_{ijk}(\chi^{*i})\bar{l}_n^{Lj}(l_5^L)^{ck}. \tag{3.42}$$

Introducing the flavor vector for the negatively charged leptons

$$l^- = (e_1, e_2, e_3, E_2, E_3, E_4, F_4, E_5), \tag{3.43}$$

the (TeV scale) mass matrix for the negatively charged leptons at this point reads

$$M_{l^-}^{\text{TeV}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y_{28}u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y_{38}u \\ Y_{41}u & Y_{42}u & Y_{43}u & Y_{44}u & Y_{45}u & Y_{46}u & Y_{47}u & Y_{48}u \\ Y_{51}u & Y_{52}u & Y_{53}u & Y_{54}u & Y_{55}u & Y_{56}u & Y_{57}u & Y_{58}u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Y_{68}u \\ Y_{71}u & Y_{72}u & Y_{73}u & Y_{74}u & Y_{75}u & Y_{76}u & Y_{77}u & Y_{78}u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.44}$$

As it appears in this matrix, the degeneracy of the 0 eigenvalue is greater than three, meaning that the number of the degrees of freedom that should acquire mass after the electroweak symmetry breaking is more than the three SM's charged leptons. Thus, because the spectrum should contain no light particles apart from the SM ones, we have to get rid of such presence. To do so, we identify the

[10]The Yukawa terms that can be built with the sextet lead to Majoranna masses for the exotic neutral leptons N_5 and N_1. They are of the form $S\bar{l}_5(l_5^c + l_1)$.

E_5^{+L} with the charge conjugate of the right handed component of E_4^-. As a result, the right handed partner of E_4^- should belong to the lepton triplet l_5^{cL} ($(E_4^{-R})^c \sim (1, 3, \frac{1}{3})$) rather than being a singlet[11]. The spectrum thus contains three EW mass (SM) plus four TeV mass exotic charged leptons.

With this assumption, the most general Yukawa Lagrangian containing all gauge invariant lepton-lepton-scalar terms responsible for the charged lepton masses after the two stages of SSB reads

$$\mathcal{L}_Y^{l^-} = \left[\bar{l}_n^L \left(y_{ni}\chi + k_{ni}\eta\right) + \bar{l}_1^L h_{1_i}\rho^*\right] l_i^R + \bar{l}_n^L l_1^L C_{n1} S_c + \bar{l}_n^L l_5^{cL} \left(Y_{n5}\chi^* + K_{n5}\eta^* + C_{n5} S_c\right), \tag{3.45}$$

where $n = 2, 3, 4$ and

$$l_i^R = \left(e_1^{-R}, e_2^{-R}, e_3^{-R}, E_2^{-R}, E_3^{-R}, E_4^{-R}, F_4^{-R}\right).$$

y_{ni}, k_{ni} and h_{ni} are the Yukawa couplings of the left-handed n and the right-handed charged lepton fields i with the scalars χ, η and ρ, respectively.

Y_{nn} and K_{nn} are the Yukawa couplings of the left-handed lepton fields n with the scalars χ and η respectively.

C_{nn} are the Yukawa couplings of the left-handed lepton fields n with the sextet S_c.

The mass matrix of the charged leptons reads

$$M_{l^-} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_{11}v & h_{12}v & h_{13}v & h_{14}v & h_{15}v & 0 & h_{17}v \\ k_{21}w_2 + C_{21}c_1 & k_{22}w_2 & k_{23}w_2 & k_{24}w_2 & k_{25}w_2 & -Y_{26}u - k_{26}w_3 + C_{26}c_1 & k_{27}w_2 \\ k_{31}w_2 + C_{31}c_1 & k_{32}w_2 & k_{33}w_2 & k_{34}w_2 & k_{35}w_2 & -Y_{36}u - k_{36}w_3 + C_{36}c_1 & k_{37}w_2 \\ y_{41}u + y_{41}u + k_{41}w_3 + C_{41}c_2 & y_{42}u + k_{42}w_3 & y_{43}u + k_{43}w_3 & y_{44}u + k_{44}w_3 & y_{45}u + k_{45}w_3 & k_{46}w_2 + C_{46}c_2 & y_{47}u + k_{47}w_3 \\ y_{51}u + k_{51}w_3 + C_{51}c_2 & y_{52}u + k_{52}w_3 & y_{53}u + k_{53}w_3 & y_{54}u + k_{54}w_3 & y_{55}u + k_{55}w_3 & k_{56}w_2 + C_{56}c_2 & y_{57}u + k_{57}w_3 \\ k_{61}w_2 + C_{61}c_1 & k_{62}w_2 & k_{63}w_2 & k_{64}w_2 & k_{65}w_2 & -Y_{66}u - k_{66}w_3 + C_{66}c_1 & k_{67}w_2 \\ y_{71}u + k_{71}w_3 + C_{71}c_2 & y_{72}u + k_{72}w_3 & y_{73}u + k_{73}w_3 & y_{74}u + k_{74}w_3 & y_{75}u + k_{75}w_3 & k_{76}w_2 + C_{76}c_2 & y_{77}u + k_{77}w_3 \end{pmatrix} \tag{3.46}$$

[11] This is the only possible set for $\beta = 1/\sqrt{3}$. Any other attempt to limit the number of the extra degrees of freedom would lead to Yukawa terms that, despite being allowed by symmetry arguments, are forbidden by mass scale arguments (e.g. identifying F_4^{-L} with e_1^{-R} would lead to a TeV Yukawa term $\propto e_1^{-R} e_2^{-R}$).

As in the quark case, the diagonalization of the mass matrix before the EWSB (in the limit $v = w_2 = w_3 = c_1 = c_2 = 0$) shows that the number of the masseless charged leptons is exactly three which is the number of the SM leptons. This means that all exotic leptons acquire mass of the Λ_{NP} scale after the $SU(3)_L$ SSB. Again, this is why such heavy leptons have not been observed at the electroweak scale.

Thus, the Yukawa Lagrangian that can be built with the scalar fields discussed above meets the phenomenological expectations. In fact, it provides (heavy) masses to all exotic degrees of freedom after the $SU(3)_L$ SSB before undergoing another transition at the electroweak scale where all the remaining SM particles acquire (light) masses.

When we move to the mass basis, the diagonalization of the up-type, down-type and lepton mass matrices is performed with a singular value decomposition (SVD) where each matrix requires two unitary matrices that rotate the interaction and mass eigenstates. We perform the bi-unitary transformation on both M_q and M_l

$$V^{(q)^\dagger} M_q W^{(q)} = M'^{(q)}, \qquad U^{(l)^\dagger} M_l W^{(l)} = M'^{(l)}. \tag{3.47}$$

Here $M'^{(q)}$ and $M'^{(l)}$ are the diagonal mass matrices. $V^{(q)}$, $U^{(l)}$ and $W^{(f)}$ are unitary rotation matrices relating (unprimed) fermion interaction eigenstates and (primed) mass eigenstates

$$q^L = V^{(q)} q'^L, \quad l^L = U^{(l)} l'^L, \quad f^R = W^{(f)} f'^R. \tag{3.48}$$

They satisfy

$$\begin{aligned} V^{(q)^\dagger} M_q M_q^\dagger V^{(q)} &= W^{(q)^\dagger} M_q M_q^\dagger W^{(q)} = {M'^{(q)}}^2, \\ U^{(l)^\dagger} M_l M_l^\dagger U^{(l)} &= W^{(l)^\dagger} M_l M_l^\dagger W^{(l)} = {M'^{(l)}}^2, \end{aligned} \tag{3.49}$$

where q and l stand for the quark and the lepton fields, respectively, while f stands for all fermion fields. As a result, $M'^{(u,d)}$ and $M'^{(l)}$ are the diagonal 5×5, 4×4 and the 7×7 mass matrices for the up-type (3.39), down-type (3.40) quarks and the charged leptons (3.46), respectively.

3.1.3 Neutrino mass in the 331 model

The non-vanishing neutrino masses is an indisputable evidence of physics beyond the standard model. In the absence of any direct evidence of their mass, the observation of neutrino oscillation suggests that most likely the new physics energy range Λ is well above the electroweak scale [110–116]. Within the SM, as a masseless particle would not be able to change its chirality, neutrinos were introduced as purely masseless and strictly left-chiral fermions for which no gauge-invariant renormalizable mass term can be constructed. Consequently, in the SM there is no mixing in the lepton sector. In this way, neutrino masses and lepton mixing would be a form of manifestation of NP. As bounds on their masses are led to by decay processes of the leptons[12], their (small) values are accommodated within BSM scenarios and even in the non-minimal version of the SM by either introducing right-handed (RH) neutral singlets or breaking the lepton number $\mathcal{L}$, respectively. As any other fermion, neutrino Dirac mass terms are generated through the coupling of the left- and the (introduced) right-handed fields (1.22). They stem from the gauge-invariant Yukawa Lagrangian

$$\mathcal{L}^{\text{Yukawa}} \propto Y_\nu^{ij} \bar{L}_i^L H \nu_R^j + h.c. \overset{\text{SSB}}{\longrightarrow} \bar{\nu}_i^L M_\nu^{ij} \nu_j^R + h.c., \quad \text{with} \quad M_\nu^{ij} = \frac{h}{\sqrt{2}} Y_\nu^{ij}, \tag{3.50}$$

wher h is the *vev* of the scalar mutiplet of the BSM theory. Since they have no electric charge, neutrino masses have been able to be generated, however, without the addition of their right-handed partners. In fact, within the SM, it was noted by Weinberg [119] and inpendently by Wilczek and Zee [120] that small Majoranna neutrinos masses could be generated through the dimension-five effective operator

$$\mathcal{O}_\nu = \overline{L_{ia}^{cL}} L_{jb}^L \Phi^k \Phi^l \left(f_{ij} \varepsilon_{ak} \varepsilon_{bl} \right), \tag{3.51}$$

[12]The study of electron energy spectrum in tritium β-decay, based on the analysis of the Kurie plot, has led to the bound $m_{\nu_e} < 2.2$ eV (95%C.L) [117]. Upper bounds on the masses ν_μ and ν_τ are $m_{\nu_\mu} < 170$ keV (90%C.L) and $m_{\nu_\tau} < 18.2$ MeV (95%C.L) [118]. They are obtained from the decays of π-mesons and τ-leptons, respectively.

where i, j are flavor indices, a, b, l, k are isospin indices and f_{ij} are couplings of the order of Λ^{-1}. This is the only dimension-five operator allowed by the SM's gauge symmetry. Neutrino masses and mixings are generated when the neutral component of the scalar doublet Φ develops its *vev*. The neutrino mass matrix elements will read

$$M_\nu^{ij} = \frac{f_{ij}\langle\Phi\rangle^2}{\Lambda}. \tag{3.52}$$

Small Majoranna masses are thus generated through the see-saw mechanism if $\Lambda \sim 10^9 - 10^{13}$ GeV [4]. The realization of this approach can easily be implemented in the 331 model as well. In fact, it has been shown that seesaw neutrino masses could be generated from an effective dimension-five operator that is built of the leptonic and the scalar triplets [121–123]. Moreover, within the context of 331 models with RH neutrinos, small neutrino masses can also be generated. Models where the lepton triplets are of the form $(\nu, l, \nu^c)^L$, with the RH neutrinos $\nu^{cR} = (\nu^c)^L$ introduced as the third component of the triplets, have been extensively studied [124–126]. Neutrino masses are generated at the tree level with the three Higgs triplets. The neutrino spectrum, however, shows to be unrealistic as there is only one squared mass-splitting. In fact, the spectrum contains three Dirac fields with one massless and two degenerate in mass $\sim h^\nu v/\sqrt{2}$, with h^ν being the Yukawa coupling and v the electroweak-scale *vev*, and massless Majoranna fields ν^L and ν^R. The addition of the Higgs sextet, however, proves able to generate small neutrino masses via a type-II seesaw mechanism [127] and provides a possible explanation of the large splitting $\Delta m^2_{\text{atm}} \gg \Delta m^2_{\text{sol}}$ with no need of fine-tuning [128,129]. This could be realized within our framework since neutral leptons interaction terms with the scalar sextet could be generated. However, as the current study relies mostly on the unitarity feature of the mixing matrix of charged leptons with (massive) neutral ones, and not on their actual masses (see below), the exploration of the neutrino mass spectrum is left out for future consideration.

3.2 Interaction terms with the gauge bosons and LFUV

As stated before, lepton flavor universality violation arises from the different couplings of the lepton fields with the model's gauge bososn. The interaction terms of the fermionic and bosonic fields are derived from the covariant derivatives (3.25) acting on the various fields. The kinetic part of the Lagrangian for the left-handed triplets is

$$\begin{aligned}\mathcal{L}^{3}_{\text{kin.}} =&\bar{\psi}_L i\gamma^\mu D_\mu \psi_L\\ =&\bar{\psi}_L i\gamma^\mu \left(\partial_\mu - igW^a_\mu \hat{T}^a - ig_X X X_\mu \hat{T}^9\right)\psi_L,\end{aligned} \tag{3.53}$$

where $\psi_L \in \left\{Q^L_3, l^L_n, l^L_4, l^L_5\right\}$, $(n = 2, 3)$. We denote with ψ^i_3 the three entries of the triplet ψ_L where $i = 1, 2, 3$ indicate the up-type, down-type and the third entries, respectively. the kinetic part of the Lagrangian for the left-handed antitriplets is

$$\begin{aligned}\mathcal{L}^{\bar{3}}_{\text{kin.}} =&\psi_L i\gamma^\mu D_\mu \bar{\psi}_L\\ =&\psi_L i\gamma^\mu \left(\partial_\mu + igW^a_\mu (\hat{T}^a)^T - ig_X X X_\mu \hat{T}^9\right)\bar{\psi}_L,\end{aligned} \tag{3.54}$$

where $\bar{\psi}_L \in \left\{Q^L_m, l^L_1\right\}$, $(m = 1, 2)$. We denote with $\psi^i_{\bar{3}}$ the three entries of the anti-triplet ψ_L where $i = 1, 2, 3$ indicate the up-type, down-type and the third entries, respectively. As for the right-handed singlets, the kinetic part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{kin.}} =&\psi_R i\gamma^\mu D_\mu \bar{\psi}_R\\ =&\bar{\psi}_R i\gamma^\mu \left(\partial_\mu - ig_X X X_\mu \hat{T}^9\right)\psi_R,\end{aligned} \tag{3.55}$$

The interaction terms of the various fermionic currents with the the four neutral Z_μ, A_μ, Z'_μ and V^0_μ gauge bosons of the theory are shown in table (3.1), where the gauge fields are given in Eqs. (3.32) and (3.34), and the relations between the two coupling constants g and g_X and the two mixing angles θ_{331} and $\theta_{\mathcal{W}}$ are given in Eq. (3.36).

The interaction terms of the various fermionic currents with the two charged $W^\pm_\mu$

Fermionic current	$A_\mu \times g\sqrt{3}\cos\theta_{331}\cos\theta_{\mathcal{W}}\times$	$Z_\mu \times \dfrac{g}{2\cos\theta_{\mathcal{W}}}\times$	$Z'_\mu \times \dfrac{g}{2\sqrt{3}\cos^2\theta_{\mathcal{W}}\sin\theta_{331}}\times$	$V^0_\mu \times \dfrac{g}{\sqrt{2}}\times$
$\bar{\psi}^1_3\gamma^\mu\psi^1_3$	$\left[\frac{2}{3}+X_{(3)}\right]$	$\left[1-\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}+2X_{(3)}\right)\right]$	$\left[-1+\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}+2X_{(3)}\right)\right]$	0
$\bar{\psi}^2_3\gamma^\mu\psi^2_3$	$\left[-\frac{1}{3}+X_{(3)}\right]$	$\left[-1+\sin^2\theta_{\mathcal{W}}\left(\frac{2}{3}-2X_{(3)}\right)\right]$	$\left[-1+\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}+2X_{(3)}\right)\right]$	0
$\bar{\psi}^3_3\gamma^\mu\psi^3_3$	$\left[-\frac{1}{3}+X_{(3)}\right]$	$\left[\sin^2\theta_{\mathcal{W}}\left(\frac{2}{3}-2X_{(3)}\right)\right]$	$\left[2-\sin^2\theta_{\mathcal{W}}\left(-\frac{8}{3}-2X_{(3)}\right)\right]$	0
$\bar{\psi}^1_{\bar{3}}\gamma^\mu\psi^1_{\bar{3}}$	$\left[-\frac{2}{3}+X_{(\bar{3})}\right]$	$\left[-1+\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}-2X_{(\bar{3})}\right)\right]$	$\left[1-\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}-\frac{2}{3}X_{(\bar{3})}\right)\right]$	0
$\bar{\psi}^2_{\bar{3}}\gamma^\mu\psi^2_{\bar{3}}$	$\left[\frac{1}{3}+X_{(\bar{3})}\right]$	$\left[1-\sin^2\theta_{\mathcal{W}}\left(\frac{2}{3}+2X_{(\bar{3})}\right)\right]$	$\left[1-\sin^2\theta_{\mathcal{W}}\left(\frac{4}{3}-\frac{2}{3}X_{(\bar{3})}\right)\right]$	0
$\bar{\psi}^3_{\bar{3}}\gamma^\mu\psi^3_{\bar{3}}$	$\left[\frac{1}{3}+X_{(\bar{3})}\right]$	$\left[-\sin^2\theta_{\mathcal{W}}\left(\frac{2}{3}+2X_{(\bar{3})}\right)\right]$	$\left[-2+\sin^2\theta_{\mathcal{W}}\left(\frac{8}{3}+2X_{(\bar{3})}\right)\right]$	0
$\bar{\psi}_R\gamma^\mu\psi_R$	X	$-2\sin^2\theta_{\mathcal{W}}X$	$2X\sin^2\theta_{\mathcal{W}}$	0
$\bar{\psi}^2_3\gamma^\mu\psi^3_3$	0	0	0	1
$\bar{\psi}^3_{\bar{3}}\gamma^\mu\psi^2_{\bar{3}}$	0	0	0	-1

Table 3.1: Couplings of the different fermionic fields with the neutral gauge bosons.

Fermionic current	$W^\pm_\mu \times \dfrac{g}{\sqrt{2}}\times$	$Y^\pm_\mu \times \dfrac{g}{\sqrt{2}}\times$
$\bar{\psi}^1_3\gamma^\mu\psi^2_3$	1	0
$\bar{\psi}^1_3\gamma^\mu\psi^3_3$	0	1
$\bar{\psi}^2_{\bar{3}}\gamma^\mu\psi^3_{\bar{3}}$	1	0
$\bar{\psi}^3_{\bar{3}}\gamma^\mu\psi^1_{\bar{3}}$	0	1

Table 3.2: Couplings of the different fermionic fields with the charged gauge bosons.

and $Y^\pm_\mu$ gauge bosons of the theory are shown in table (3.2).

3.2.1 Gauge Bosons Contributions

In the folowing, and based on the global analyses stated in Section(2.1.1), our main focus will be the vector/axial contributions which are assumed to be the larger ones in both $b \longrightarrow sll$ and $b \longrightarrow l\nu_l$. These contributions can only come from the gauge bosons, neutral and charged, of the theory. Hence, we provide, in what follows, the expressions of the couplings of the charged and the neutral gauge bosons with the fermions, where the flavor vectors are given in Eq. (3.18), expressed in the interaction eigenbasis.

Neutral currents

The couplings of fermions to neutral gauge bosons Z_μ, A_μ, Z'_μ and V^0_μ ($W^{6,7}_\mu$) are given by the interaction Lagrangian density expressed in the interaction eigenbasis

$$\mathcal{L}_{\text{N.C.}} = \mathcal{L}_{Z_\mu} + \mathcal{L}_{A_\mu} + \mathcal{L}_{Z'_\mu} + \mathcal{L}_{V^0_\mu}, \tag{3.56}$$

where

$$\mathcal{L}_{A_\mu} = \sqrt{3}\cos\theta_{331}\cos\theta_{\mathcal{W}} g A_\mu \left\{ \frac{2}{3}\bar{U}\gamma^\mu U - \frac{1}{3}\bar{D}\gamma^\mu D - \bar{f}\gamma^\mu f \right\}, \tag{3.57}$$

$$\begin{aligned}\mathcal{L}_{V^{\pm 0}_\mu} = \frac{g}{\sqrt{2}} \Big\{ & V^{+0}_\mu \left[\bar{d}^L_3\gamma^\mu D^L_3 - \bar{U}^L_m\gamma^\mu u^L_m - \bar{N}^L_1\gamma^\mu \nu^L_1 + \bar{e}^{-L}_n\gamma^\mu E^{-L}_n + \bar{E}^{-L}_4\gamma^\mu F^{-L}_4 + \bar{N}^L_5\gamma^\mu P^L_5 \right] \\ & + V^{-0}_\mu \left[\bar{D}^L_3\gamma^\mu d^L_3 - \bar{u}^L_m\gamma^\mu U^L_m - \bar{\nu}^L_1\gamma^\mu N^L_1 + \bar{E}^{-L}_n\gamma^\mu e^{-L}_n + \bar{F}^{-L}_4\gamma^\mu E^{-L}_4 + \bar{P}^L_5\gamma^\mu N^L_5 \right] \Big\},\end{aligned} \tag{3.58}$$

with $V^{\pm 0}_\mu$ being a combination of the two neutral W^6_μ and W^7_μ as shown in Eq. (3.27).

$$
\begin{aligned}
\mathcal{L}_{Z_\mu} =& g\cos\theta_{\mathcal{W}} Z_\mu \Bigg\{ \bar{U}_L\gamma^\mu \begin{pmatrix} \frac{1-\cos^2\theta_{331}}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1-\cos^2\theta_{331}}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\cos^2\theta_{331}}{2} & 0 & 0 \\ 0 & 0 & 0 & -2\cos^2\theta_{331} & 0 \\ 0 & 0 & 0 & 0 & -2\cos^2\theta_{331} \end{pmatrix} U_L \\
&+ \bar{D}_L\gamma^\mu \begin{pmatrix} -\frac{1+\cos^2\theta_{331}}{2} & 0 & 0 & 0 \\ 0 & -\frac{1+\cos^2\theta_{331}}{2} & 0 & 0 \\ 0 & 0 & -\frac{1+\cos^2\theta_{331}}{2} & 0 \\ 0 & 0 & 0 & \cos^2\theta_{331} \end{pmatrix} D_L \\
&+ \bar{f}_L^-\gamma^\mu \begin{pmatrix} \frac{-1+3\cos^2\theta_{331}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1+3\cos^2\theta_{331}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1+3\cos^2\theta_{331}}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\cos^2\theta_{331} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\cos^2\theta_{331} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1+3\cos^2\theta_{331}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3\cos^2\theta_{331} \end{pmatrix} f_L^- , \\
&+ \bar{f}_R^-\gamma^\mu \begin{pmatrix} 3\cos^2\theta_{331} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3\cos^2\theta_{331} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3\cos^2\theta_{331} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\cos^2\theta_{331} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\cos^2\theta_{331} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-3\cos^2\theta_{331}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3\cos^2\theta_{331} \end{pmatrix} f_R^- \\
&+ \left(\frac{1+3\cos^2\theta_{331}}{2}\right) \bar{N}_L\gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} N_L + \cos^2\theta_{331}\left(-2\bar{U}_R\gamma^\mu U_R + \bar{D}_R\gamma^\mu D_R\right) \Bigg\}
\end{aligned}
\tag{3.59}
$$

and

$$
\begin{aligned}
\mathcal{L}_{Z'_\mu} = \frac{\cos\theta_{331}}{g_X} Z'_\mu \Bigg\{ & \bar{U}_L \gamma^\mu \begin{pmatrix} \frac{9g^2+g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & \frac{9g^2+g_X^2}{3\sqrt{6}} & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}}g^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{-18g^2+g_X^2}{3\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & \frac{-18g^2+g_X^2}{3\sqrt{6}} \end{pmatrix} U_L \\
& + \bar{D}_L \gamma^\mu \begin{pmatrix} \frac{9g^2+g_X^2}{3\sqrt{6}} & 0 & 0 & 0 \\ 0 & \frac{9g^2+g_X^2}{3\sqrt{6}} & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}}g^2 & 0 \\ 0 & 0 & 0 & \sqrt{6}g^2 \end{pmatrix} D_L \\
& + \bar{f}_L^- \gamma^\mu \begin{pmatrix} \frac{9g^2-g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-9g^2-2g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-9g^2-2g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{18g^2-2g_X^2}{3\sqrt{6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{18g^2-2g_X^2}{3\sqrt{6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-9g^2-g_X^2}{3\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{18g^2-2g_X^2}{3\sqrt{6}} \end{pmatrix} f_L^- \\
& + \bar{f}_R^- \gamma^\mu \begin{pmatrix} -\frac{g_X^2}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{g_X^2}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g_X^2}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{g_X^2}{\sqrt{6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{g_X^2}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-9g^2+g_X^2}{3\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{g_X^2}{\sqrt{6}} \end{pmatrix} f_R^- \\
& + \bar{N}_L \gamma^\mu \begin{pmatrix} \frac{9g^2-g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{9g^2+2g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{9g^2+2g_X^2}{3\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-18g^2-g_X^2}{3\sqrt{6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{9g^2+2g_X^2}{3\sqrt{6}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-9g^2+g_X^2}{3\sqrt{6}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{18g^2+g_X^2}{3\sqrt{6}} \end{pmatrix} N_L \\
& + \frac{g_X^2}{3\sqrt{6}} \left(2\bar{U}_R \gamma^\mu U_R - \bar{D}_R \gamma^\mu D_R \right) \Bigg\}
\end{aligned}
\tag{3.60}
$$

Charged currents

The couplings of fermions to both the SM and the non-SM charged gauge bosons $W_\mu^\pm$ and $Y_\mu^\pm$ respectively are given by the interaction Lagrangian density expressed in the interaction eigenbasis

$$\mathcal{L}_{\text{C.C.}} = \mathcal{L}_{W_\mu^\pm} + \mathcal{L}_{Y_\mu^\pm}, \tag{3.61}$$

where

$$\begin{aligned}\mathcal{L}_{W_\mu^\pm} =& \frac{g}{\sqrt{2}} W_\mu^+ \left\{ \bar{U}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} D_L + \bar{N}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} f_L^- + \overline{(f^{-R})^c} \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} N_L \right\} \\ &+ \frac{g}{\sqrt{2}} W_\mu^- \left\{ \bar{D}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U_L + \bar{f}_L^- \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} N_L + \bar{N}_L \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} (f^{-R})^c \right\},\end{aligned} \tag{3.62}$$

and

$$\begin{aligned}\mathcal{L}_{Y_\mu^\pm} =& \frac{g}{\sqrt{2}} Y_\mu^+ \left\{ \bar{U}_L \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} D_L + \bar{N}_L \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} f_L^- + \overline{(f^{-R})^c} \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} N_L \right\} \\ &+ \frac{g}{\sqrt{2}} Y_\mu^- \left\{ \bar{D}_L \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} U_L + \bar{f}_L^- \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} N_L + \bar{N}_L \gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} (f^{-R})^c \right\},\end{aligned} \tag{3.63}$$

3.2.2 Mixing matrices in the 331 model

When we move to the mass basis by rotating the fermion fields (Eq. (3.48)), flavor-violating factors arise as in the SM. In fact, the two rotation matrices for the left-handed quarks $V^{(u),(d)}$ and the two for the left-handed leptons $U^{(l)}$ and $U^{(\nu)}$ will lead to the appearance of mixing matrices in both the quark and the lepton sectors.

Mixing in the quark sector

From Eq. (3.62), the mixing matrix CKM for the 331 model is given by the W_μ^+ coupling to the quarks

$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{U}_L \gamma^\mu \varphi D_L = \frac{g}{\sqrt{2}} W_\mu^+ \bar{U}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} D_L, \tag{3.64}$$

with the 5×4 matrix $V_{\text{CKM}}^{331} = V^{(u)^\dagger} \varphi V^{(d)}$

$$V_{\text{CKM}}^{331} = \begin{pmatrix} V_{11}^{*(u)} & V_{21}^{*(u)} & V_{31}^{*(u)} & V_{41}^{*(u)} & V_{51}^{*(u)} \\ V_{12}^{*(u)} & V_{22}^{*(u)} & V_{32}^{*(u)} & V_{42}^{*(u)} & V_{52}^{*(u)} \\ V_{13}^{*(u)} & V_{23}^{*(u)} & V_{33}^{*(u)} & V_{43}^{*(u)} & V_{53}^{*(u)} \\ V_{14}^{*(u)} & V_{24}^{*(u)} & V_{34}^{*(u)} & V_{44}^{*(u)} & V_{54}^{*(u)} \\ V_{15}^{*(u)} & V_{25}^{*(u)} & V_{35}^{*(u)} & V_{45}^{*(u)} & V_{55}^{*(u)} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_{11}^{(d)} & V_{12}^{(d)} & V_{13}^{(d)} & V_{14}^{(d)} \\ V_{21}^{(d)} & V_{22}^{(d)} & V_{23}^{(d)} & V_{24}^{(d)} \\ V_{31}^{(d)} & V_{32}^{(d)} & V_{33}^{(d)} & V_{34}^{(d)} \\ V_{41}^{(d)} & V_{42}^{(d)} & V_{43}^{(d)} & V_{44}^{(d)} \end{pmatrix}, \tag{3.65}$$

whose elements can be written as

$$\left(V_{\text{CKM}}^{331}\right)_{k,l} = \sum_{n=1,2,3} V_{nk}^{*(u)} V_{nl}^{(d)}, \tag{3.66}$$

where $n = 1, 2, 3$ are the SM mass indices and $k = 1, .., 5$ and $l = 1, .., 4$ are quark flavor indices. If we consider only the flavor subspace of the SM particles and remain at low energies, i.e. limit the number of indices to three for both the up and down-quarks, we would recover the unitary V_{CKM} matrix of the SM.

Mixing in the lepton sector

For the leptons, the PMNS matrix is generated by the coupling of W_μ^+ with the leptons. From Eq. (3.62)

$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{N}_L \gamma^\mu \xi f_L^- = \frac{g}{\sqrt{2}} W_\mu^+ \bar{N}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} f_L^- , \tag{3.67}$$

where the flavor vectors f_L^- and N_L read

$$\begin{aligned} f_L^- &= \left(e_1^{-L}, e_2^{-L}, e_3^{-L}, E_4^{-L}, F_4^{-L}, E_2^{-L}, E_3^{-L}\right)^T , \\ N_L &= \left(\nu_1^L, \nu_2^L, \nu_3^L, N_4^L, N_5^L, P_5^L, N_1^L\right)^T . \end{aligned} \tag{3.68}$$

The 7×7 U_{PMNS}^{331} would be built by the combination $U_{\text{PMNS}}^{331} = U^{(\nu)\dagger} \xi U^{(l)}$

$$U_{\text{PMNS}}^{331} = \begin{pmatrix} U_{11}^{*(\nu)} & U_{21}^{*(\nu)} & U_{31}^{*(\nu)} & \dots & U_{71}^{*(\nu)} \\ U_{12}^{*(\nu)} & U_{22}^{*(\nu)} & U_{32}^{*(\nu)} & \dots & U_{72}^{*(\nu)} \\ U_{13}^{*(\nu)} & U_{23}^{*(\nu)} & U_{33}^{*(\nu)} & \dots & U_{73}^{*(\nu)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{17}^{*(\nu)} & U_{27}^{*(\nu)} & U_{37}^{*(\nu)} & \dots & U_{77}^{*(\nu)} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} U_{11}^{(l)} & U_{12}^{(1)} & U_{13}^{(l)} & \dots & U_{17}^{(1)} \\ U_{21}^{(l)} & U_{22}^{(l)} & U_{23}^{(l)} & \dots & U_{27}^{(l)} \\ U_{31}^{(l)} & U_{32}^{(l)} & U_{33}^{(l)} & \dots & U_{37}^{(l)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_{71}^{(l)} & U_{72}^{(l)} & U_{73}^{(l)} & \dots & U_{77}^{(l)} \end{pmatrix} . \tag{3.69}$$

Its elements would be expressed as

$$\left(U^{331}_{\rm PMNS}\right)_{ij} \simeq \left(U^{\rm SM}_{\rm PMNS}\right)_{ij} + U^{*(\nu)}_{4i} U^{(l)}_{4j}, \tag{3.70}$$

where

$$\left(U^{\rm SM}_{PMNS}\right)_{ij} = \sum_{n=1,2,3} U^{*(\nu)}_{ni} U^{(l)}_{nj}. \tag{3.71}$$

Here i, j are lepton generation indices. Yet, another term that contributes to the PMNS matrix in the considered 331 model results from the identification we set for the fields in order to have masses consistent with the observation. In fact, the identification of a left-handed component of a triplet with the charge conjugate of a right-handed field made this latter transform as a triplet under $SU(3)_L$, which allows for a symmetry-conserving interaction term as shown in Eq. (3.62)

$$\overline{(f^{-R})^c}\gamma^\mu \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} N_L. \tag{3.72}$$

Thus, the elements of the $(U^{331}_{\rm PMNS})$ matrix, within our framework, read

$$\left(U^{331}_{\rm PMNS}\right)_{ij} = \left(U^{\rm SM}_{\rm PMNS}\right)_{ij} + U^{*(\nu)}_{4i} U^{(l)}_{4j} + W^{*(l)}_{5i} U^{(\nu)}_{5j}. \tag{3.73}$$

The $U^{331}_{\rm PMNS}$ could be developed for a non-minimal 331 model with $\beta = 1/\sqrt{3}$ by obtaining bounds on the mixing angles in the leptonic sector studying LFV processes [130], but given the current study, exploiting the unitarity of the rotation matrices $U^{(\nu)}$ and $U^{(l)}$ proves sufficient for the current study.

3.2.3 The NP contribution and Wilson coefficients

As the theory at hand covers two widely seperated energy ranges, Λ_{SM} and Λ_{NP}, we introduce a parameter ϵ defined as

$$\epsilon = \frac{\Lambda_{\text{EW}}}{\Lambda_{\text{NP}}}, \tag{3.74}$$

to keep track of the energy order we are dealing with. Moreover, our analysis will be focusing on the only non-SM contributions to the Wilson coefficients at the lowest order in ϵ. i.e. the 3×3 SM block matrices. The Wilson coefficients of interest will be $\mathcal{C}_9^{(\prime)}$, $\mathcal{C}_{10}^{(\prime)}$ and $\mathcal{C}_{V_L}$, which contribute to the $(V - A)$ operators $\mathcal{O}_9^{(\prime)ij}$, $\mathcal{O}_{10}^{(\prime)ij}$ and $\mathcal{O}_{V_L}^{ij}$ defined in Eqs. (2.16), (2.20) and (2.22), respectively. To track the mixing terms relevant for our study at each order of energy, we perform the diagonalization of the mass matrices (3.39), (3.40) and (3.46) order by order in ϵ. At each order, the following pattern for the mixing matrices V, U and W is observed [131]

(i) At the order ϵ^0, each of the mixing matrices consists in a 3×3 unitary block that mixes the SM particles among themselves. The elements of the matrices are of the form $V_{ni}^* V_{mj}$ with n, m being SM mass indices and i, j are flavor indices.

(ii) The $\mathcal{O}(\epsilon^1)$ correction to the rotation matrices leads to an only mixing between SM particles and exotic fields, but not SM fields alone or exotic fields alone. At that order, the matrix elements take the form $V_{ni}^* V_{mj}$ with n, m being SM an NP mass indices, respectively, and i, j are flavor indices.

(iii) At the order ϵ^2, the contributions connect all the exotic particles. The elements of mixing matrices read $V_{ni}^* V_{mj}$ with n, m being only NP mass indices, and i, j are flavor indices.

Contributions to the $b \longrightarrow s$ transition

The interactions of Z'_μ, Z_μ and A_μ with the right-handed quarks, as can be seen from Eqs. (3.60), (3.59) and (3.57), respectively, are proportional to the identity in flavor space, so no flavor change can arise at any order in ϵ. As a result, neither contributes to the Wilson coefficients $\mathcal{C}'_{9,10}$. Thus, only contributions to $\mathcal{O}_{9,10}$ are possible. As for the heavy V^0_μ ($W^{6,7}_\mu$), we see from Eq. (3.58) and Tab. (3.1) that these gauge bosons always couple an SM fermion to an exotic one in the interaction basis. Thus, when moving to the mass basis, this only occurs at $\mathcal{O}(\epsilon^1)$.

Contribution from Z_μ

The Lagrangian describing the interaction of Z_μ with all the down-type quarks and the leptons in the mass basis reads

$$\begin{aligned}\mathcal{L}_{Z_\mu} \supset g\cos\theta_{\mathcal{W}} Z_\mu \Bigg\{ &\left(\frac{-1+\cos^2\theta_{331}}{2}\delta_{kl} + \frac{1+3\cos^2\theta_{331}}{2}V^*_{4k}V_{4l}\right)\bar{D}^k_L\gamma^\mu D^l_L \\ &+\left(\frac{-1+3\cos^2\theta_{331}}{2}\delta_{ij} + \frac{1+3\cos^2\theta_{331}}{2}\sum_{\lambda=4,5,6}U^*_{\lambda i}U_{\lambda j}\right)\bar{f}^{i-}_L\gamma^\mu f^{j-}_L . \\ &+\left(3\cos^2\theta_{331}\delta_{ij} + \frac{1-9\cos^2\theta_{331}}{2}W^*_{6i}W_{6j}\right)\bar{f}^{i-}_R\gamma^\mu f^{j-}_R \Bigg\}\end{aligned} \tag{3.75}$$

When the flavor changing transition $b \to s$ is mediated by the SM's Z_μ, we see that the interaction arises at $\mathcal{O}(\epsilon^2)$, where a V_{CKM} matrix element mixing the fourth heavy state to a quark flavor state appears when we move to the mass basis. The low-energy $\mathcal{O}(\epsilon^0)$ SM matrix is, however, diagonal in the flavor space as is shown in Eq. (3.59).

Contribution from Z'_μ

When we move to the mass eigenbasis, the Lagrangian describing the interaction of Z'_μ with all the down-type quarks and the leptons reads

$$\begin{aligned}\mathcal{L}_{Z'_\mu} \supset \frac{\cos\theta_{331}}{g_X} Z'_\mu \Bigg\{ &\left(\frac{9g^2+g_X^2}{3\sqrt{6}}\delta_{kl} + \frac{-18g^2-g_X^2}{3\sqrt{6}}V^*_{3k}V_{3l} + \frac{9g^2-g_X^2}{3\sqrt{6}}V^*_{4k}V_{4l}\right)\bar{D}^k_L\gamma^\mu D^l_L \\ &+\left(\frac{-9g^2-2g_X^2}{3\sqrt{6}}\delta_{ij} + \frac{18g^2+g_X^2}{3\sqrt{6}}U^*_{1i}U_{1j} + \frac{9g^2}{\sqrt{6}}\sum_{\lambda=4,5,6}U^*_{\lambda i}U_{\lambda j}\right)\bar{f}^{i-}_L\gamma^\mu f^{j-}_L , \\ &+\left(\frac{-g_X^2}{\sqrt{6}}\delta_{ij} + \frac{-9g^2+4g_X^2}{3\sqrt{6}}W^*_{6i}W_{6j}+\right)\bar{f}^{i-}_R\gamma^\mu f^{j-}_R \Bigg\}\end{aligned} \tag{3.76}$$

where all the fermion fields here are massive, k, l and i, j are flavor indices for quarks and leptons, respectively. It is clear that the flavor changing transition $b \to s$ mediated by the heavy Z'_μ arises already at low energy $\mathcal{O}(\epsilon^0)$ where the V and W connect only SM fermions among themselves. In fact, as the restriction of the interaction matrix in Eq. (3.60) to the SM particles is not proportional to the identity in flavor space, a V_{CKM} element $V^*_{3k}V_{3l}$ appears in the Lagrangian when we move to the mass basis, mixing the third SM mass and a quark flavor eigenstates, with $k = 1, .., 5$ for the up-type and $l = 1, .., 4$ for the down-type quarks.

Contributions to $b \longrightarrow c$ transition

The flavor changing transition $b \longrightarrow c$ is mediated by the two charged gauge bosons of the theory: the SM's (light) W^+_μ and the (heavy) Y^+_μ bosons.

Contribution from W^+_μ

In the mass basis, the Lagrangian describing the interaction of W^+_μ with the massive fermions reads

$$\begin{aligned}\mathcal{L}_{W^+_\mu} = \frac{g}{\sqrt{2}} W^+_\mu \Big\{ &\sum_{n=1,2,3} V^{*(u)}_{nk} V^{(d)}_{nl} (\bar{u}^L_k \gamma^\mu d^L_l) + \sum_{n=1,2,3} U^{*(\nu)}_{ni} U^{(l)}_{nj} (\bar{\nu}^L_i \gamma^\mu l^L_j) \\ &+ U^{*(\nu)}_{4i} U^{(l)}_{4j} (\bar{N}^L_4 \gamma^\mu E^L_4) + W^{*(l)}_{5i} U^{(\nu)}_{5j} (\overline{(E^{-R}_4)^c} \gamma^\mu N^L_5) \Big\}\end{aligned}, \quad (3.77)$$

where $n = 1, 2, 3$ are SM mass indices, k, l and i, j are quark and lepton flavor indices, respectively. It is clear that the charged quark flavor-changing transition $d_l \longrightarrow u_k$ occurs already at $\mathcal{O}(\epsilon^0)$ as a V_{CKM} matrix element $\sum_{n=1,2,3} V^{*(u)}_{nk} V^{(d)}_{nl}$ appears when we switch to the mass basis. Thus, no NP contribution arises at this order. As for the leptons, the leading non-SM contribution arises at $\mathcal{O}(\epsilon^2)$, where the lepton mixing matrices couple an exotic (heavy) mass and flavor eigentates.

Contribution from Y^+_μ

The Lagrangian describing the interaction of Y^+_μ with the massive fermions reads

$$\begin{aligned}\mathcal{L}_{Y^+_\mu} = \frac{g}{\sqrt{2}} Y^+_\mu \Big\{ & \Big(V^{*(u)}_{4k} V^{(d)}_{1l} + V^{*(u)}_{5k} V^{(d)}_{2l} + V^{*(u)}_{3k} V^{(d)}_{4l} \Big) (\bar{u}^L_k \gamma^\mu d^L_l) \\ & + (U^{*(\nu)}_{4i} U^{(l)}_{1j} + U^{*(\nu)}_{2i} U^{(l)}_{4j} + U^{*(\nu)}_{3i} U^{(l)}_{5j} + U^{*(\nu)}_{5i} U^{(l)}_{7j}) (\bar{\nu}^L_i \gamma^\mu l^L_j) , \\ & + W^{*(l)}_{6i} U^{(\nu)}_{6j} (\overline{(E^{-R}_4)^c} \gamma^\mu P^L_5) \Big\} \end{aligned} \tag{3.78}$$

For the heavy gauge boson, the flavor-changing quark transition does not occur at $\mathcal{O}(\epsilon^0)$. It arises, however, at $\mathcal{O}(\epsilon^1)$, where Y^+_μ always couples an SM particle with an exotic one.

In what follows, we write the leading order effective Hamitonian for these flavor-changing processes mediated by the model's gauge bosons.

NP contribution to the $\mathcal{C}_9$ and $\mathcal{C}_{10}$ Wilson coefficients

For the flavor-changing neutral process $b \longrightarrow s$, the transition, mediated by the SM's gauge boson Z_μ arises at $\mathcal{O}(\epsilon^2)$, and because there is no $\mathcal{O}(\epsilon^2)$ suppression due to the gauge boson's mass, the leading NP contribution for the Z_μ is also at $\mathcal{O}(\epsilon^2)$. Thus, we consider only $\mathcal{O}(\epsilon^0)$ SM terms of the lepton sector. The transition mediated by the heavy Z'_μ boson, on the other hand, starts at $\mathcal{O}(\epsilon^0)$, but due to the $\mathcal{O}(\epsilon^2)$ suppression that results from the heavy mass of the Z'_μ boson, we conclude that the NP contribution from the Z'_μ boson starts at $\mathcal{O}(\epsilon^2)$. Thus, as in the case of Z_μ, we consider only $\mathcal{O}(\epsilon^0)$ SM terms of the lepton sector. In the case of heavy neutral gauge boson V^0_μ ($W^{6,7}_\mu$), we have seen that the $b \longrightarrow s$ transitions arises at $\mathcal{O}(\epsilon^1)$. Moreover, since these processes are mediated by the heavy gauge boson, an additionnal $\mathcal{O}(\epsilon^2)$ leads to an overall $\mathcal{O}(\epsilon^3)$ suppression. As a result, the $W^{6,7}_\mu$ contribution can be neglected compared to Z'_μ and Z_μ's.

We can thus rewrite equation (3.76) at the order of interest as

$$\begin{aligned}\mathcal{L}_{Z_\mu} = g\cos\theta_{\mathcal{W}} Z_\mu \Bigg\{ & \left(\frac{1+3\cos^2\theta_{331}}{2} V^*_{4k}V_{4l}\right)\left(\bar{d}^l_L\gamma^\mu d^k_L\right) \\ & +\frac{1}{2}\left(\frac{-1+9\cos^2\theta_{331}}{2}\delta_{ij}\right)\left(\bar{l}^i\gamma^\mu l^j\right) \\ & +\frac{1}{2}\left(\frac{3\cos^2\theta_{331}+1}{2}\delta_{ij}\right)\left(\bar{l}^i\gamma^\mu\gamma_5 l^j\right)\Bigg\}\end{aligned} \tag{3.79}$$

The (leading order) $\mathcal{O}(\epsilon^2)$ effective Hamiltonian for the flavor-changing neural transition mediated by the Z_μ reads[13]

$$\mathcal{H}^{Z_\mu}_{\text{eff}} \supset \frac{\cos^2\theta_{\mathcal{W}}\left(1+3\cos^2\theta_{331}\right)}{8}\frac{g^2}{M_Z^2}\frac{4\pi}{\alpha}V^*_{4k}V_{4l}\delta_{ij}\left[\left(-1+9\cos^2\theta_{331}\right)\mathcal{O}^{ijkl}_9+\left(1+3\cos^2\theta_{331}\right)\mathcal{O}^{ijkl}_{10}\right] \quad , \tag{3.80}$$

where $\alpha = e^2/4\pi$ is the fine structure constant and the operators $\mathcal{O}^{ijkl}_{9,10}$ are defined in Eq. (2.16), corresponding to the $(\bar{d}^k d^l)(\bar{l}^i l^j)$ flavor structure.

Following the above considerations, we eliminate the coupling g by means of Eq. (3.36). Equation (3.76) can thus be rewritten as

$$\begin{aligned}\mathcal{L}_{Z'_\mu} = \frac{\cos\theta_{331}}{g_X} Z'_\mu \Bigg\{ & \left(-\frac{g_X^2}{3\sqrt{6}\cos^2\theta_{331}}V^*_{3k}V_{3l}\right)\left(\bar{d}^l_L\gamma^\mu d^k_L\right) \\ & +\frac{1}{2}\left(-\frac{g_X^2}{3\sqrt{6}\cos^2\theta_{331}}\right)\left(\frac{1+9\cos^2\theta_{331}}{2}\delta_{ij}-U^*_{1i}U^*_{1j}\right)\left(\bar{l}^i\gamma^\mu l^j\right) \\ & +\frac{1}{2}\left(-\frac{g_X^2}{3\sqrt{6}\cos^2\theta_{331}}\right)\left(\frac{-1+3\cos^2\theta_{331}}{2}\delta_{ij}+U^*_{1i}U^*_{1j}\right)\left(\bar{l}^i\gamma^\mu\gamma_5 l^j\right)\Bigg\}\end{aligned} \tag{3.81}$$

[13] $V^*_{(3,4)k}V_{(3,4)l} \equiv V^{(d)*}_{(3,4)k}V^{(d)}_{(3,4)l}$ with (d) stands for a down-type quark, and $U^*_{1i}U_{1j} \equiv U^{(l)*}_{1i}U^{(l)}_{1j}$ with the superscript (l) stands for a charged lepton.

The leading-order $\mathcal{O}(\epsilon^2)$ effective Hamiltonian for the FCNC transition mediated by the Z'_μ in terms of the effective operators is then

$$\mathcal{H}_{\text{eff}}^{Z'_\mu} \supset \frac{g_X^2}{108\cos^2\theta_{331}} \frac{1}{M_{Z'_\mu}^2} V_{3k}^* V_{3l} \frac{4\pi}{\alpha} \left\{ \left[\left(\frac{1+9\cos^2\theta_{331}}{2} \right) \delta_{ij} - U_{1i}^* U_{1j} \right] \mathcal{O}_9^{ijkl} + \left[\left(\frac{3\cos^2\theta_{331}-1}{2} \right) \delta_{ij} + U_{1i}^* U_{1j} \right] \mathcal{O}_{10}^{ijkl} \right\}. \tag{3.82}$$

By matching $\mathcal{H}_{\text{eff}}^{Z'_\mu}$ and $\mathcal{H}_{\text{eff}}^{Z_\mu}$ onto Eq. (2.19), the NP contributions to the Wilson coefficients can be written in terms of the quantities $f_{Z'}$ and f_Z as

$$\mathcal{C}_9^{ij} = f_{Z'} \left(-\lambda_{ij} + \frac{1+3\tan^2\theta_{\mathcal{W}}}{2} \delta_{ij} \right) + f_Z \left(-1 + 3\tan^2\theta_{\mathcal{W}} \right) \delta_{ij}, \tag{3.83}$$

and

$$\mathcal{C}_{10}^{ij} = f_{Z'} \left(\lambda_{ij} + \frac{\tan^2\theta_{\mathcal{W}} - 1}{2} \delta_{ij} \right) + f_Z \left(1 + \tan^2\theta_{\mathcal{W}} \right) \delta_{ij}, \tag{3.84}$$

where

$$f_{Z'} = -\frac{1}{2\sqrt{2} G_F V_{tb} V_{ts}^*} \frac{4\pi}{\alpha} \frac{1}{6 - 2\tan^2\theta_{\mathcal{W}}} \frac{g^2}{M_{Z'}^2} V_{3k}^* V_{3l}, \tag{3.85}$$

and

$$f_Z = -\frac{1}{2\sqrt{2} G_F V_{tb} V_{ts}^*} \frac{4\pi}{\alpha} \frac{1}{8} \frac{g^2}{M_Z^2} V_{4k}^* V_{4l}, \tag{3.86}$$

with $\lambda_{ij} = U_{1i}^* U_{1j}$. Here we have eliminated $\cos\theta_{331}$ and the coupling g_X by means of Eq. (3.36). Despite the fact that our model allows for lepton flavor violating transitions with different leptons in the final state ($i \neq j$), these processes have not been observed yet, so, assuming that they are suppressed, we set their coefficients to zero. The solution $f_{Z'} = 0$, i.e. the NP contribution is zero, should be discarded as it would mean the absence of LFUV. We are left, thus, with $\lambda_{ij} = 0$ for $i \neq j$. By definition

$$\lambda_{ij} = 0 \Longrightarrow U_{1i}^* U_{1j} = 0. \tag{3.87}$$

Equation (3.87) does not necessarily imply that both U matrix elements have to be zero; one rotation matrix entry can be non-zero for a generation i (e.g. $i = 1$) while the other two entries (e.g. $j = 2, 3$) are zero and yet ensuring the above annihilation. We denote with I the generation for which the entry for the rotation matrix is non-zero, and with i the other generations. We get

$$\begin{aligned} \mathcal{C}_9^I &= f_{Z'}\left(-\lambda_I + \frac{1+3\tan^2\theta_{\mathcal{W}}}{2}\right) + f_Z\left(-1+3\tan^2\theta_{\mathcal{W}}\right), \\ \mathcal{C}_{10}^I &= f_{Z'}\left(\lambda_I + \frac{\tan^2\theta_{\mathcal{W}}-1}{2}\right) + f_Z\left(1+\tan^2\theta_{\mathcal{W}}\right), \end{aligned} \tag{3.88}$$

and

$$\begin{aligned} \mathcal{C}_9^i &= f_{Z'}\left(\frac{1+3\tan^2\theta_{\mathcal{W}}}{2}\right) + f_Z\left(-1+3\tan^2\theta_{\mathcal{W}}\right), \\ \mathcal{C}_{10}^i &= f_{Z'}\left(\frac{\tan^2\theta_{\mathcal{W}}-1}{2}\right) + f_Z\left(1+\tan^2\theta_{\mathcal{W}}\right). \end{aligned} \tag{3.89}$$

Inverting relations (3.89) we get

$$f_{Z'} = \frac{1+\tan^2\theta_{\mathcal{W}}}{4\tan^2\theta_{\mathcal{W}}}\mathcal{C}_9^i - \frac{-1+3\tan^2\theta_{\mathcal{W}}}{4\tan^2\theta_{\mathcal{W}}}\mathcal{C}_{10}^i. \tag{3.90}$$

From the system of equations (3.89) and (3.88) we get

$$2\lambda_I f_{Z'} = \mathcal{C}_{10}^I - \mathcal{C}_9^I - \mathcal{C}_{10}^i + \mathcal{C}_9^i. \tag{3.91}$$

We now have to identify which index corresponds to which lepton, knowing that based on phenomenological constraints, the electronic NP contribution to the effective Hamiltonian $C_{9,10}^e$ is absent.

(i) If we identify the electron with the index i (for which the entry for the rotation matrix vanishes), we set $\mathcal{C}_{9,10}^i = 0$. Equation (3.90) implies that $f_{Z'} = 0$, solution that has to be discarded since it would mean no LFUV.

(ii) If the electron is identified with the index I, the coefficients $\mathcal{C}_{9,10}^I$ must be set to zero and the remaining index i would correspond to the other two generations.

In this case, Eqs. (3.91) and (3.90) yield constraints on the non-vanishing NP Wilson coefficients for μ and τ

$$\frac{\mathcal{C}_9^\mu}{\mathcal{C}_{10}^\mu} = \frac{2\tan^2\theta_{\mathcal{W}} + \lambda_e\,(1 - 3\tan^2\theta_{\mathcal{W}})}{2\tan^2\theta_{\mathcal{W}} - \lambda_e\,(1 + \tan^2\theta_{\mathcal{W}})}. \tag{3.92}$$

Due to the unitarity of the 7×7 rotation matrix U, we have

$$\lambda_I =\mid U_{1I} \mid^2 = 1 - \sum_{(i\neq I)i=1}^{7} \mid U_{1i} \mid^2, \tag{3.93}$$

which means that $0 < \lambda_e \leq 1$. Imposing that $\mid \mathcal{C}_{10}^\mu \mid \leq \mid \mathcal{C}_9^\mu \mid$, the one-dimensional scenario of the global analysis that favors NP in $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$ within the 1σ interval $[-0.75, -0.49]$ [86] is explained for $0.71 \leq \lambda_e \leq 0.86$ ($\theta_{\mathcal{W}} \simeq 29°$), where the best-fit point presented in Tab. (2.1) corresponds to $\lambda_e \equiv |U_{1e}|^2 = 0.68$. The exact equality $\mathcal{C}_9^\mu/\mathcal{C}_{10}^\mu = -1$, obtained for $\lambda_e = 1$, which means that the left-handed interaction of the electron is a mass eigenstate, is also allowed for this case. It is worth mentioning that in case A of the set with $\beta = -1/\sqrt{3}$ [131], the allowed region to the Wilson coefficients imposing that $\mid \mathcal{C}_9^\mu \mid \leq \mid \mathcal{C}_{10}^\mu \mid$ is explained for $0.81 \leq \lambda_e^L \leq 1$, which agrees with our set, provided that $\mid \mathcal{C}_{10}^\mu \mid$ is supposed to be less than $\mid \mathcal{C}_9^\mu \mid$ ($\mathcal{C}_{10}^\mu = -\mathcal{C}_9^\mu$), only for $\lambda_e = 1$. The case B in Ref. [131], however, is not taken into account within our model since the right-handed components are not concerned with the modification applied to the fields. The other two scenarios (NP in $\mathcal{C}_{9,10}^\mu = -\mathcal{C'}_{9,10}^\mu$ and NP in $\mathcal{C}_9^\mu$ [86]) cannot be described in the framework of our model. In fact, since no FCNC arises for right-handed quarks due to their diagonal interaction terms in flavor space (Eqs. (3.59) and (3.60)), $\mathcal{C'}_{9,10}^\mu = 0$. Figure (3.1) shows the allowed region for the Wilson coefficients as indicated by the gray wedge imposing $\mathcal{C}_9^\mu/\mathcal{C}_{10}^\mu$ to remain between -1 and -2.04. The light grey wedge are the results obtained for $0.71 \leq \lambda_e \leq 1$. The dark gray wedge represents the the one-dimensional scenario of the global analysis that favors NP in $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$ within the 1σ interval $[-0.75, -0.49]$.

In summary, the electron (first generation of SM leptons) has to be identified with

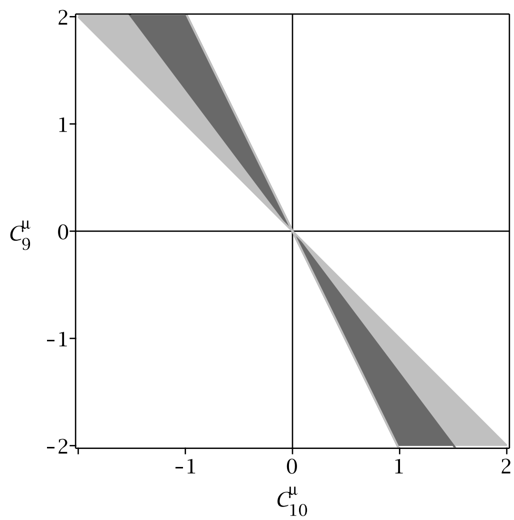

Figure 3.1: Allowed regions for the Wilson coefficients for $\beta = 1/\sqrt{3}$ imposing that $\mid \mathcal{C}_{10}^{\mu} \mid \leq \mid \mathcal{C}_{9}^{\mu} \mid$. The dark gray wedge shows the favored one-dimensional NP scenario in $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$.

the non-vanishing entry in the rotation matrix U in order to have non-vanishing NP contributions to Wilson coefficients, for both μ and τ that agree with the favored one-dimensional scenario of NP in $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$.

NP contribution to the $\mathcal{C}_{V_L}$ Wilson coefficient

When considering only contributions at the lowest order in ϵ, we have seen that the $b \to c$ transition mediated by the SM gauge boson $W_\mu^\pm$ occurs at $\mathcal{O}(\epsilon^0)$ whereas the leading order non-SM contribution for leptons arises at $\mathcal{O}(\epsilon^2)$. We conclude that the leading order effective Hamiltonian describing $b \longrightarrow c$ process mediated by $W_\mu^\pm$ is $\mathcal{O}(\epsilon^2)$. For the heavy gauge boson, on the other hand, the flavor-changing quark transition arises at $\mathcal{O}(\epsilon^1)$. In addition to that, the Hamiltonian contains a $\mathcal{O}(\epsilon^2)$ suppression (compared to the SM) that comes from the heavy mass of the gauge boson in the propagator. Therefore, the Y_μ^+ contribution starts at an already high overall $\mathcal{O}(\epsilon^3)$ order which can be neglected compared to the $\mathcal{O}(\epsilon^2)$ contribution from the W_μ^+ gauge boson regardless of what the order of the leading contribution of the leptons might be. The (leading order) $\mathcal{O}(\epsilon^2)$ effective Hamiltonian for the flavor changing charged transition mediated by the W_μ^+ is thus

$$\mathcal{H}_{\text{eff}}^{W_\mu^+} = \frac{g^2}{2M_W^2} \sum_{n=1,2,3} (V_{nk}^{*(u)} V_{nl}^{(d)}) \left[\sum_{n=1,2,3} (U_{ni}^{*(\nu)} U_{nj}^{(l)}) + U_{4i}^{*(\nu)} U_{4j}^{(l)} + U_{5i}^{*(\nu)} W_{5j}^{(l)} \right] (\bar{u}_k^L \gamma_\mu d_l^L)(\bar{\nu}_i^L \gamma^\mu l_j^L), \tag{3.94}$$

where $n = 1, 2, 3$ are SM mass indices, k, l and i, j are quark and lepton flavor indices, respectively. For the charged transition of interest ($k = 3$ and $l = 2$), $\sum_{n=1,2,3}(V_{nk}^{*(u)} V_{nl}^{(d)})$ is nothing but the V_{CKM} element V_{cb} (Eq. (3.66)) and $\sum_{n=1,2,3}(U_{ni}^{*(\nu)} U_{nj}^{(l)})$ is the element $(U_{\text{PMNS}})_{ij}^{\text{SM}}$ (Eq.(3.71)). Thus, exploiting the unitarity of the SM's U_{PMNS} matrix at the order of interest for $i = j$

$$\mathcal{H}_{\text{eff}}^{W^+} = \frac{g^2}{2M_W^2} V_{cb} \left[1 + U_{4i}^{*(\nu)} U_{4i}^{(l)} + U_{5i}^{*(\nu)} W_{5i}^{(l)} \right] (\bar{c}_L \gamma_\mu b_L)(\bar{\nu}_L \gamma^\mu l_L). \tag{3.95}$$

Comparing to equation (2.23), the model's contribution to the operator $\mathcal{O}_{V_L}^{ii}$ is thus

$$\mathcal{C}_{V_L} = U_{4i}^{*(\nu)} U_{4i}^{(l)} + U_{5i}^{*(\nu)} W_{5i}^{(l)}. \tag{3.96}$$

Both 7×7 rotation matrices $U^{(\nu)}$ and $U^{(l)}$ are unitary, and so is their product, so we can write for a specific SM generation I

$$U_{4I}^{*(\nu)} U_{4I}^{(l)} = 1 - \sum_{(J \neq I) J=1}^{7} U_{4J}^{*(\nu)} U_{4J}^{(l)}. \tag{3.97}$$

The same can be said about the product 7×7 of the two unitary matrices $U^{(\nu)}$ and $W^{(l)}$

$$U_{5I}^{*(\nu)} W_{5I}^{(l)} = 1 - \sum_{(J \neq I) J=1}^{7} U_{5J}^{*(\nu)} W_{5J}^{(l)}. \tag{3.98}$$

J represents all the remaining generations. We denote with $\Lambda_I = U_{4I}^{*(\nu)} U_{4I}^{(l)}$ and $\Delta_I = U_{5I}^{*(\nu)} W_{5I}^{(l)}$. Equations (3.97) and (3.98) mean that Λ_I and Δ_I should belong each to $[0, 1[$, so $\Lambda_I + \Delta_I$ should also remain within the interval $[0, 1[$ in order for the term to account as a contribution which agrees with the allowed 1σ range for the effective coefficient $\mathcal{C}_{V_L}$ (Eq. (2.24)) for $\Lambda_I + \Delta_I \in [0.09, 0.13]$. Thus, our model provides a good explanation of the NP contribution to the $b \to cl\bar{\nu}_l$ transitions provided that the dominant contributions come from the gauge bosons rather than the Higgs sector [132].

Conclusion

Indications of LFUV in the rare flavor-changing processes $B \longrightarrow K^{(*)}l^+l^-$ and $B \longrightarrow D^{(*)}l\bar{\nu}_l$ that have been reported by the experimental collaborations of LHCb, Belle and BaBar, have triggered a large interest in possible NP interpretations since the universality of the weak interactions is one of the key predictions of the SM. The fact that the deviations from the SM expectations had been observed only in the decay of the B mesons, a speculation about a possible NP that couples mainly to the third generation of quarks and leptons would be the way to go. In fact, whether the investigation is carried out through the decay of electroweak gauge bosons, the leptonic and semileptonic decays of mesons with light quarks, or the decay of quarkonia, no deviation had been observed in the different probes performed to test this property.
Within a model-independent approach, such deviations can be computed theoretically separating short- and long-distance contributions using an effective Hamiltonian. Assuming that NP originates at a scale $\Lambda_{\rm NP} \sim \mathcal{O}(\text{TeV})$ to solve the hierarchy problem, its contributions are encoded in short-distance Wilson coefficients that factor the dimension-six semi-leptonic operators that dominate the transitions. At the low energy scale $\mu \sim m_b$, the derivation of the Wilson coefficients proceeds through two steps. The first step consists of determining them at the high scale $\mu \sim M_{W,t}$ through matching the full SM result (observable) onto the effective one and thus determining the initial conditions, while the second consists of running the Wilson coefficients down to the low-energy scale by calculating the anomalous dimension matrix needed for the solution of the regularization group equation (RGE). The global analyses for the neutral current

(NC) anomaly $R^{\mu/e}_{K^{(*)}}$ agree in favor of a large shift in the two Wilson coefficients $\mathcal{C}^{\mu(')}_9$ and $\mathcal{C}^{\mu(')}_{10}$, either separately or in pairs, which factor the operators $\left(\bar{s}_{L(R)}\gamma_\mu b_{L(R)}\right)(\bar{\mu}\gamma^\mu\mu)$ and $\left(\bar{s}_{L(R)}\gamma_\mu b_{L(R)}\right)(\bar{\mu}\gamma^\mu\gamma_5\mu)$, respectively, with an absence of NP contribution to any electronic Wilson coefficient. While the charged current (CC) anomaly $R^{\tau/l}_{D^{(*)}}$, is described by a shift in the Wilson coefficient $\mathcal{C}_{VL}$ that factors the dimension-six operator $(\bar{c}_L\gamma_\mu b_L)(\bar{\tau}\gamma^\mu\nu_\tau)$.

In order to provide a simultaneous dynamical explanation for these deviations, a scenario that embeds a Z' model, widely used in the literature, is proposed. Based on the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, 331 models is a BSM theory from which the SM gets recovered at low energies as the models' gauge symmetry gets broken down spontaneously at a high energy scale, and then eventually to $U(1)_{em}$ at the electroweak scale. As the minimal versions of these models do not generate LFUV since they have to obey gauge-anomaly cancellation, a non-minimal 331 model is adopted. In fact, from the requirement that the lepton generations should not be embedded equally into $SU(3)_L$ representation in order for LFUV to arise from different couplings between the leptons and the gauge bosons, two additional lepton triplets had to be introduced. Moreover, in order to accommodate the experimental observations, one of the fermion fields had to be identified with the charge conjugate of the right handed component of another generation, which not only helps reducing the number of the additional degrees of freedom, but also prevents the existence of unwanted non-SM light particles in the final spectrum. Choosing the specific value for the parameter $\beta = 1/\sqrt{3}$ to ensure non-exotic charges for both SM and new fields in the spectra, we investigated the ability of this model to reproduce the deviations observed in both FCCC and FCNC transitions assuming that these latter are dominated by the model's charged and neutral gauge bosons, respectively. In other words, we worked out how this model could accommodate the favored solutions of the global analyses performed within a model-independent approach.

In the case of the neutral transition $b \longrightarrow sl^+l^-$, the adopted model turned out to accommodate significant NP contribution to the two Wilson coefficients $\mathcal{C}^\mu_9$ (positive) and $\mathcal{C}_{10}^{\mu}$ (negative) in agreement with NP scenarios favored by global fits with the assumption of the absence of the electronic contribution in $b \longrightarrow se^+e^-$ as it has not been observed. Furthermore, a prediction for the values of both Wilson coefficients $\mathcal{C}_9$ and $\mathcal{C}_{10}$ for $b \longrightarrow s\tau^+\tau^-$ could be made from the electronic and muonic ones. The model, however, does not account for any contribution to the Wilson coefficients $\mathcal{C'}_{9,10}$ as it has no LFUV right-handed currents. Moreover, lepton flavor transition might arise in our model, which is a frequent feature of models generating LFUV couplings [133]. However, the non-observation of lepton flavor violation (LFV) $b \longrightarrow sl_il_j$ allowed us to set constraints on the mixing matrices between the mass and the interaction fermion eigenstates. Even though not (yet) observed, a prediction concerning $b \longrightarrow s\bar{\nu}\nu$ transition Wilson coefficients could be made in our framework which would arise at the same order in ϵ as other FCNC $b \longrightarrow sl^+l$ transitions.

In the case of the hints of LFUV observed in charged transitions $b \longrightarrow cl\nu_l$, namely $R_{D^{(*)}}$, once again, the non-observation of lepton flavor violation (LFV) $b \longrightarrow cl\bar{\nu'}$ allowed us to set constraints on the mixing matrices between the mass and the interaction fermion eigenstates. The model thus proved able to explain the dominance of the vector/axial exchange which is favored by global fits. Moreover, the analysis showed that the observed deviations in $b \longrightarrow c$ transitions could be explained by the exchange of the SM's W_μ^+ but not the heavy Y_μ^+ as this latter couples with the fermions at a high order in ϵ ($\mathcal{O}(\epsilon^2)$) and is suppressed furthermore by the heavy mass of the heavy mediator. It turns out that since in the mass basis, the light $W_\mu^\pm$ bosons have diagonal couplings in the SM subspace, at the energy range of interest, LFUV appears only due to mixing effects in the lepton sector which result from the masses of the neutrinos. More precisely, the leading order contribution stems from the PMNS matrix element that mixes a lepton with a massive neutrino without which, such contributing term would not appear.

Even though, for the problem at hand, exploiting the unitarity of the rotation matrices $U^{(\nu)}$ and $U^{(l)}$ has proved sufficient in explaining the B-anomalies observed in the charged current transitions, exploring the neutrino mass spectrum is of paramount importance and requires an accurate analysis within our framework that should be considered in future work.

Appendix A

OPE and short-distance QCD effects

The effective Hamitonian, including QCD effects, is generalized to

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_n V^n_{\text{CKM}}\, \mathcal{C}_n(\mu) \mathcal{O}_n(\mu), \tag{A.1}$$

where V^n_{CKM} denotes the CKM structure of the operator $\mathcal{O}_n$. The amplitude for the decay of a meson $M = K, D, B, ...$ to a final state F is obtained by the projection of the Hamilton operator onto the external states

$$\begin{aligned} i\mathcal{A}_{\text{eff}}(M \to F) &= \langle F | \mathcal{H}_{\text{eff}} | M \rangle \\ &= \frac{G_F}{\sqrt{2}} \sum_n V^n_{\text{CKM}}\, \mathcal{C}_n(\mu) \left\langle F | \mathcal{O}_n(\mu) | M \right\rangle , \end{aligned} \tag{A.2}$$

For definiteness, we consider the non-leptonic quark-level decay $c \longrightarrow su\bar{d}$. The tree-level W-exchange amplitude for this decay (without QCD effects) is

$$i\mathcal{A}^{(0)}_{\text{eff}}(c \longrightarrow su\bar{d}) = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} \left[\mathcal{C}_2(M_W) \left\langle \mathcal{O}_2 \right\rangle_T \right], \tag{A.3}$$

where $\mathcal{C}_2(M_W) = 1$ and $\langle \mathcal{O}_2 \rangle_T$ is the tree-level matrix element of $\mathcal{O}_2$ with

$$\mathcal{O}_2 = (\bar{s}_L^\alpha \gamma^\mu c_L^\alpha)\left(\bar{u}_L^\beta \gamma_\mu c_L^\beta\right). \tag{A.4}$$

Taking into account QCD corrections, the (generalized) effective Hamitonian constructed to reproduce the amplitude $\mathcal{A}$ in the full theory reads

$$\mathcal{H}_{\text{eff}}^{1-\text{loop}}(c \longrightarrow su\bar{d}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\mathcal{C}_1 \mathcal{O}_1 + \mathcal{C}_2 \mathcal{O}_2\right], \tag{A.5}$$

with

$$\mathcal{O}_1 = \left(\bar{s}_L^\alpha \gamma^\mu c_L^\beta\right)\left(\bar{u}_L^\beta \gamma_\mu c_L^\alpha\right). \tag{A.6}$$

Here α and β are color indices and $\mathcal{O}_1$ is a newly generated operator that has the same flavor form of $\mathcal{O}_2$ but different color structure. The Wilson coefficients $\mathcal{C}_n$ are obtained by *matching* the full theory onto the effective one by requiring

$$\mathcal{A}_{\text{eff}}(c \longrightarrow su\bar{d}) = \mathcal{A}_{\text{full}}(c \longrightarrow su\bar{d}), \tag{A.7}$$

where

$$i\mathcal{A}_{\text{eff}}(c \longrightarrow su\bar{d}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\mathcal{C}_1(\mu)\langle \mathcal{O}_1 \rangle + \mathcal{C}_2(\mu)\langle \mathcal{O}_2 \rangle\right] \tag{A.8}$$

with μ being a renormalization scale. The full amplitude for the $c \longrightarrow su\bar{d}$ decay is obtained by evaluating the diagrams (a)-(c) in Fig. (A.1). To $\mathcal{O}(\alpha_s)$, it is found to be [70] (taking $m_q^2 \ll p^2 \lesssim M_W^2$)

$$\begin{aligned} i\mathcal{A}_{\text{full}}^{1-\text{loop}} &= i\mathcal{A}_{(\text{a})}^{1-\text{loop}} + i\mathcal{A}_{(\text{b})}^{1-\text{loop}} + i\mathcal{A}_{(\text{c})}^{1-\text{loop}} \\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[-3\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} \langle \mathcal{O}_1 \rangle_T + \left(1 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2}\right)\langle \mathcal{O}_2 \rangle_T\right], \end{aligned} \tag{A.9}$$

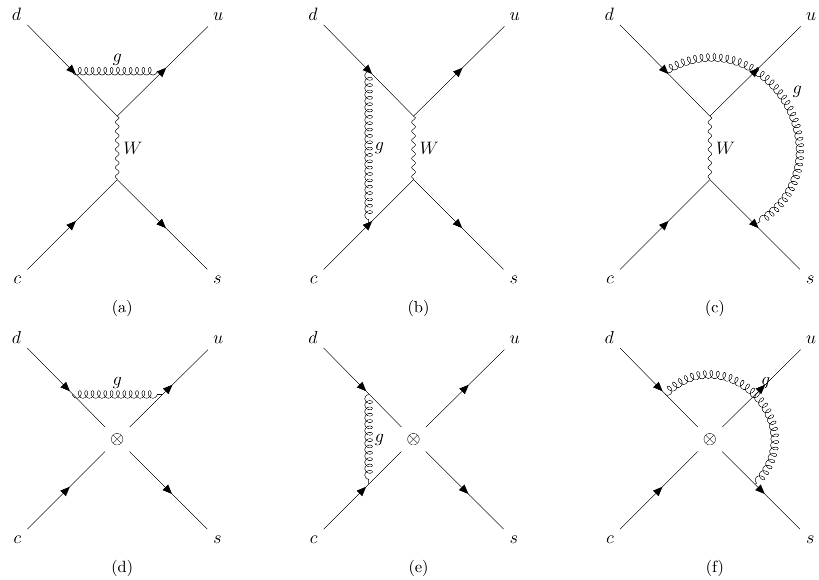

Figure A.1: One loop current-current diagrams for the non-leptonic weak decay $c \longrightarrow su\bar{d}$ in the full theory: (a), (b) and (c), and in the effective theory (d), (e) and (f).

where the tree-level matrix elements of both operators $\mathcal{O}_1$ and $\mathcal{O}_2$ are

$$\begin{aligned} \langle \mathcal{O}_1 \rangle_T &= \langle F | \mathcal{O}_1 | M \rangle_T \,, \\ \langle \mathcal{O}_2 \rangle_T &= \langle F | \mathcal{O}_2 | M \rangle_T \,. \end{aligned} \tag{A.10}$$

$\alpha_s = g_s^2/4\pi$ where g_s is the QCD coupling, N being the number of colors and p is the off-shell momentum carried by all the massless external quark lines, which is taken to be not too far from M_W. In the limit $\mu \to p$, the matching procedure yields

$$\begin{cases} \mathcal{C}_1(\mu) = 0 - 3\dfrac{\alpha_s}{4\pi} \ln \dfrac{M_W^2}{\mu^2} \\ \mathcal{C}_2(\mu) = 1 + \dfrac{3}{N}\dfrac{\alpha_s}{4\pi} \ln \dfrac{M_W^2}{\mu^2} \end{cases} . \tag{A.11}$$

Thus, the whole calculation reduces to an effective tree-level calculation with effective couplings that have to be deduced by comparing both theories.

To the same order in α_s, the resulting expressions for the current-current matrix elements $\langle \mathcal{O}_n \rangle$ are found to remain divergent, even after *quark field renormalization* [70]. Consequently, an *operator renormalization* is required. The renormalization amounts to inserting the Wilson coefficients $\mathcal{C}_n(\mu)$ in the vertex for the one loop current-current diagrams (d), (e) and (f) in computing the effective amplitude. This would provide a factorization of both contributions: the short-distance (Wilson coefficients) and the long-distance (operator matrix elements), which constitutes the most important feature of the OPE. Hereby, the factorization amounts to splitting the logarithm according to

$$\ln \frac{M_W^2}{-p^2} = \ln \frac{M_W^2}{\mu^2} + \ln \frac{\mu^2}{-p^2}, \tag{A.12}$$

which means that the scale μ which was the infra-red (IR) cut-off in the full theory becomes the ultra-violet (UV) cut-off in the effective one. As a consequence, the one-loop effective amplitude reads

$$\begin{aligned} i\mathcal{A}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Bigg\{ & \mathcal{C}_1(\mu) \left[\left(1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) \langle \mathcal{O}_1 \rangle_T + \left(-3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) \langle \mathcal{O}_2 \rangle_T \right] \\ & + \mathcal{C}_2(\mu) \left[\left(-3 \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) \langle \mathcal{O}_1 \rangle_T + \left(1 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} \right) \langle \mathcal{O}_2 \rangle_T \right] \Bigg\}, \end{aligned} \tag{A.13}$$

from which we read off the operator renormalization matrix

$$Z_{nm} = \mathbb{1} + \frac{\alpha_s}{4\pi} \begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}. \tag{A.14}$$

which exhibits how the operators *mix under renormalization*. In fact, the matrix structure of the renormalization "constant" shows that the Wilson coefficient $\mathcal{C}_n$ can generate the structure of $\mathcal{O}_n$ at both the tree and the one-loop levels, while $\mathcal{C}_{1(2)}$ can generate the structure of $\mathcal{O}_{2(1)}$ at the one-loop level. The operators are then generated with the anomalous dimension matrix defined in Eq. (2.9). By demanding the independence of the amplitude of μ, we find the RGE of the $\mathcal{C}_n$ (2.10) which describes the evolution of the Wilson coefficients from $\mathcal{O}(100 \text{ GeV})$ down to $\mathcal{O}(1 \text{ GeV})$ energy range.

Appendix B

Bounds on the scale of NP

Bounds on the scale of new physics can be obtained from precise experimental information which, in all cases, requires that the size of the NP amplitude cannot exceed that of the SM short-distance contribution.

B.1 From $\Delta F = 1$ processes

Some of the most stringent constraints on NP models are placed by the process $b \longrightarrow s\gamma$ from the computation of its Feynman graph

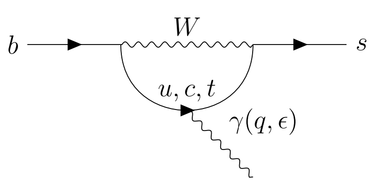

Figure B.1: Feynman diagram for $b \longrightarrow s\gamma$.

$$\mathcal{A} \sim eF_{\mu\nu}\bar{s}\sigma^{\mu\nu}\left(\frac{1+\gamma_5}{2}\right)b\frac{m_b}{M_W^2}\frac{g^2}{16\pi^2}V_{ts}^*V_{tb}F\left(\frac{m_t^2}{M_W^2}\right), \tag{B.1}$$

where $g^2/16\pi^2$ is the obvious loop-factor, e is the photon coupling constant, and m_b is the mass of the b quark which appears due to flipping its chirality. The term $V_{ts}^*V_{tb}F\left(m_t^2/M_W^2\right)$ contains the suppression factor $V_{ts}^*V_{tb}$ which translates into the domination of the running of the top quark in the loop (GIM mechanism), and a function $F\left(m_t^2/M_W^2\right)$ which is expected to be of order 1 [94].

The NP contribution to this process can be modeled by adding a six-dimension[1] operator to the Lagrangian

$$\delta\mathcal{L} = \frac{\mathcal{C}}{\Lambda^2} eF_{\mu\nu}\bar{q_L}\sigma^{\mu\nu}b_R\; H = \frac{\mathcal{C}}{\Lambda^2}\frac{v}{\sqrt{2}} eF_{\mu\nu}\bar{s_L}\sigma^{\mu\nu}b_R, \tag{B.2}$$

where the coefficient of the operator $\mathcal{C}$ is of order 1. When this term's contribution to the amplitude is compared to the SM's, and is required to be less than 10%, we get roughly the bound

$$\frac{\mathcal{A}^{\rm NP}_{\Delta F=1}}{\mathcal{A}^{\rm SM}_{\Delta F=1}} \sim \frac{\frac{\mathcal{C}}{\Lambda^2}\frac{v}{\sqrt{2}}}{\frac{m_b}{M_W^2}\frac{g^2}{16\pi^2}V_{tb}V_{ts}^*} \lesssim 0.1 \quad \rightarrow \quad \Lambda \gtrsim 70 \text{ TeV}, \tag{B.3}$$

which is clearly higher than any existing (or planned) particle accelerator facility! This bound could be softened if we take $\mathcal{C} \ll 1$ and consider non-generic NP flavor structure. However, to avoid the *flavor problem*, the effective couplings should be of order 1 in order to maintain the same flavor structure as the that of the SM. The question of what (factor) would make this coefficient smaller in order to bring the NP scale down to experimental reach while maintaining the SM's generic flavor structure is the motivation behind the MFV principle.

B.2 From $\Delta F = 2$ processes

Within the SM, the amplitudes of the mixing of B_d (and B_s and K^0) with its anti-particle[2], denoted $\Delta F = 2$ amplitudes, are generated by box-diagrams of the type in Fig. (B.2). The SM short-distance contribution to the amplitude is highly suppressed

[1] $[\mathcal{L}] = E^{+4}$, the field strength $[F_{\mu\nu}] = E^{+2}$, the fermionic fields $[\Psi] = E^{3/2}$ and the scalar field $[H] = E^{+1}$.

[2] $B_d = d\bar{b}$, $B_s = s\bar{b}$ and $K^0 = d\bar{s}$.

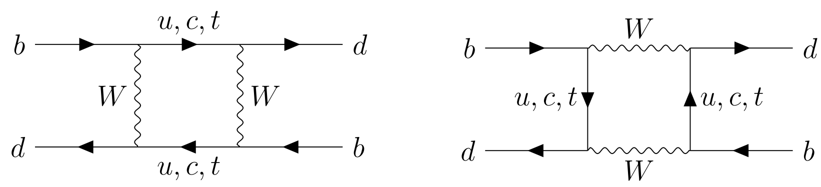

Figure B.2: Box diagrams contributing to B_d-$\bar{B}_d$ mixing.

by both the GIM mechanism (top quark running in the loop) and the hierarchical structure of the CKM matrix elements

$$\mathcal{A}^{\rm SM}_{\Delta F=2} = \frac{G_F^2 m_t^2}{16\pi^2} \left(V_{ti}^* V_{tj}\right)^2 \left\langle \bar{M} \left| \left(\bar{Q_L^i}\gamma^\mu Q_L^j\right)^2 \right| M \right\rangle \times F\left(\frac{m_t^2}{M_W^2}\right), \tag{B.4}$$

where i, j are flavor indices of the meson valence quarks, $M = K^0, B_d, B_s$ and F is a loop function of $\mathcal{O}(1)$. The NP contribution at the tree-level to the meson-antimeson mixing amplitude is modeled by the effective Lagrangian

$$\mathcal{L}_{\rm NP} = \sum_{i \neq j} \frac{\mathcal{C}^{ij}}{\Lambda^2_{\rm NP}} \left(\bar{Q_L^i}\gamma^\mu Q_L^j\right)^2, \tag{B.5}$$

where $\mathcal{C}^{ij}$ are dimensionless couplings of the dimension-six operators. The $M - \bar{M}$ mixing amplitude is thus

$$\begin{aligned} \mathcal{A}_{\Delta F=2} &= \mathcal{A}^{\rm SM}_{\Delta F=2} + \mathcal{A}^{\rm NP}_{\Delta F=2} \\ &= \left[\frac{y_t^2 \left(V_{ti}^* V_{tj}\right)^2}{16\pi^2 v^2} + \frac{\mathcal{C}^{ij}}{\Lambda^2_{\rm NP}}\right] \left\langle \bar{M} \left| \left(\bar{Q_L^i}\gamma^\mu Q_L^j\right)^2 \right| M \right\rangle \\ &= \mathcal{M}^{\rm SM}_{\Delta F=2} \left[1 + \frac{\mathcal{C}^{ij}}{y_t^2 \lambda_t^2} \left(\frac{\Lambda_{\rm SM}}{\Lambda_{\rm NP}}\right)^2\right], \end{aligned} \tag{B.6}$$

where $G_F/\sqrt{2} = 8g^2/M_W^2$, $M_W = gv/2$, $m_t = y_t v/\sqrt{2}$ and $\lambda_t = V_{ti}^* V_{tj}$. $\Lambda_{\rm SM}$ is the effective scale of the SM: $\Lambda_{\rm SM} = 4\pi v \approx 3$ TeV.

From the experimental requirement $|\mathcal{M}^{\rm NP}_{\Delta F=2}| < |\mathcal{M}^{\rm SM}_{\Delta F=2}|$ we get

$$\Lambda_{\rm NP} > \frac{3 \text{ TeV}}{\lambda_t/\sqrt{\mathcal{C}^{ij}}} \sim \begin{cases} 10^3 \text{ TeV} \times \sqrt{\mathcal{C}^{sd}} & \text{from} \quad K^0 - \bar{K}^0, \\ 10^2 \text{ TeV} \times \sqrt{\mathcal{C}^{bd}} & \text{from} \quad B_d - \bar{B}_d, \\ 10^1 \text{ TeV} \times \sqrt{\mathcal{C}^{bs}} & \text{from} \quad B_s - \bar{B}_s. \end{cases} \tag{B.7}$$

A more detailed list of the bounds derived from $\Delta F = 2$ is reported in [91] where are quoted the bounds for sets of dimension-six operators that are present in the SM, and others which arise in specific SM extensions. As a result, NP models at the TeV scale with a generic flavor structure $\mathcal{C}^{ij} = \mathcal{O}(1)$ are ruled out, otherwise, physics beyond the SM would have to be highly non generic! This (flavor) problem can be remedied with the introduction of the MFV.

Appendix C

Partial decay rate for $B \longrightarrow D\tau\bar{\nu}_\tau$

The double differential decay rate of $B \longrightarrow D\tau\bar{\nu}_\tau$ can be written as [134]

$$\frac{d\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2}{(2\pi)^3}|V_{cb}|^2 \frac{1}{16m_B^2}|\overrightarrow{p}| \left(1 - \frac{m_l^2}{q^2}\right) L_{\mu\nu}H^{\mu\nu}, \tag{C.1}$$

where $L_{\mu\nu}$ and $H^{\mu\nu}$ are the leptonic and hadronic current tensors, θ is the angle between D and l three-momenta in $(l - \bar{\nu}_\tau)$ rest frame, $\overrightarrow{p}$ is the three-momentum of D $\left(|\overrightarrow{p}| = \lambda^{1/2}\left(m_B^2, m_M^2, q^2\right) \text{ with function } \lambda\left(a, b, c\right) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca\right)$ and q^2 is the momentum transfer squared bounded at $m_l^2 \leqslant q^2 \leqslant (m_B - m_D)^2$.

C.1 Kinematics

In the rest frame of the B meson with D moving in the positive z-direction, the momenta of B , D and the virtual W^* can be written, respectively, as

$$p_B^\mu = (m_B, 0, 0, 0)\,, \qquad p_D^\mu = (E_D, 0, 0, |\overrightarrow{p}|)\,, \qquad q^\mu = (q_0, 0, 0, -|\overrightarrow{p}|)\,. \tag{C.2}$$

Three of the four polarization vectors of W^* are conveniently chosen to be orthogonal to its momentum. They are given by [135]

$$\begin{gathered}\varepsilon_\mu(\lambda_W=0)=\frac{1}{\sqrt{q^2}}\left(|\overrightarrow{p}|,0,0,-q_0\right), \qquad \varepsilon_\mu(\lambda_W=t)=\frac{1}{\sqrt{q^2}}\left(q_0,0,0,|\overrightarrow{p}|\right), \\ \varepsilon(\lambda_W=\pm)=\frac{1}{\sqrt{2}}\left(0,\pm1,-i,0\right), \end{gathered} \tag{C.3}$$

where q_0 is the energy of the W^* in the B rest frame. It reads

$$q_0=m_B-E_D \;=\; \frac{m_B^2-m_D^2+q^2}{2m_B}. \tag{C.4}$$

In the $(l-\bar{\nu}_\tau)$ center-of-mass frame, the four-momenta of the leptons are given by

$$p_l^\mu=\left(E_l,|\overrightarrow{p}_l|\sin\theta,0,|\overrightarrow{p}_l|\cos\theta\right), \qquad p_{\nu_l}^\mu=\left(|\overrightarrow{p}_l|,-|\overrightarrow{p}_l|\sin\theta,0,-|\overrightarrow{p}_l|\cos\theta\right), \tag{C.5}$$

where $E_l=\left(q^2+m_l^2\right)/2\sqrt{q^2}$ and $|\overrightarrow{p}_l|=\left(q^2-m_l^2\right)/2\sqrt{q^2}$ are the energy and the magnitude of the three-momentum of the charged lepton, respectively. In this frame, the polarization vectors of the virtual W take the form [136]

$$\varepsilon_\mu(\lambda_W=0)=(0,0,0,1), \qquad \varepsilon_\mu(\lambda_W=t)=(1,0,0,0), \qquad \varepsilon(\lambda_W=\pm)=\frac{1}{\sqrt{2}}\left(0,\mp1,-i,0\right). \tag{C.6}$$

C.2 Helicity amplitudes

The lepton-hadron correlation function is defined by the contraction of the two current tensors $L_{\mu\nu}$ and $H^{\mu\nu}$. It can be written as [134]

$$\begin{aligned} L_{\mu\nu}H^{\mu\nu} &= L^{\mu'\nu'}g_{\mu'\mu}g_{\nu'\nu}H^{\mu\nu} \\ &= \sum_{m,m',n,n'}\left(L^{\mu'\nu'}\varepsilon_{\mu'}(m)\varepsilon^*_{\nu'}(n)\right)\left(H^{\mu\nu}\varepsilon^*_\mu(m')\varepsilon_\nu(n')\right)g_{mm'}g_{nn'} \end{aligned} \tag{C.7}$$

where we have used the completeness relation for the polarization four-vectors

$$\sum_{m,m'=0,t,\pm} \varepsilon_\mu(m)\varepsilon^*_\nu(m')g_{mm'} = g_{\mu\nu}. \tag{C.8}$$

Each of the two factors in Eq. (C.7) are Lorentz-invariant. They can thus be evaluated in different Lorentz frames. The hadronic part will be evalutaed in the B rest frame, whereas the leptonic part will be evaluated in the $l - \bar{\nu}_l$ center-of-mass frame (W^* rest frame). The lepton-hadron correlation function can be written very compactly by expanding the leptonic tensor in terms of a set of Wigner's d^J-function as [134, 137]

$$\begin{aligned} L_{\mu\nu}H^{\mu\nu} =& \frac{1}{8} \sum_{\lambda_l,\lambda,\lambda_W,\lambda'_W,J,J'} (-1)^{J+J'} |h^l_{\lambda_l,\lambda_{\bar{\nu}_l}}|^2 \delta_{\lambda-\lambda_W,\lambda-\lambda'_W} \times \\ & d^J_{\lambda_W,\lambda_l-\lambda_{\bar{\nu}_l}}(\theta)\ d^{J'}_{\lambda'_W,\lambda_l-\lambda_{\bar{\nu}_l}}(\theta)\ H_{\lambda\lambda_W} H^*_{\lambda\lambda_W}, \end{aligned} \tag{C.9}$$

where

- J and J' run over 1 and 0, $\lambda_{\bar{\nu}_l} = \frac{1}{2}$ and $\lambda_W = 0(J=0), \pm, 0(J=1)$, whereas, in the rest frame of W^* ($\overrightarrow{q} = \overrightarrow{0}$), the time component transforms as $J = 0$.

- $H_{\lambda\lambda_W} \equiv H^\mu(\lambda)\varepsilon_\mu(\lambda_W)$ is the hadronic helicity amplitude which describes, for the $B \longrightarrow Dl\bar{\nu}_l$ transition, the decay of a pseudo-scalar meson (helicity $\lambda = 0$) into another pseudo-scalar meson and the four helicity states of the leptonic pair ($W_{\rm off-shell}$).

- $|h^l_{\lambda_l,\lambda_{\bar{\nu}_l}}|^2$ are the moduli squared of the helicity amplitudes evaluated in the $(l-\nu_l)$ c.m frame (where the polarization four vectors are given by Eq. (C.6). They are obtained to be [134, 136, 137]

$$\begin{aligned} |h^l_{-1/2,1/2}|^2 &= 8\left(q^2 - m_l^2\right), \qquad \text{for the non} - \text{flip transition} \\ |h^l_{1/2,1/2}|^2 &= 8\frac{m_l^2}{2q^2}\left(q^2 - m_l^2\right), \qquad \text{for the flip transition} \end{aligned} \tag{C.10}$$

Using the standard convention for Wigner's d^J-functions [16]

$$d^J_{mm'}(\theta) = \sqrt{(J+m')!\,(J-m')!\,(J+m)!\,(J-m)!} \times \sum_s \left[\frac{(-1)^{m'-m+s}}{s!\,(J+m-s)!\,(m'-m+s)!\,(J-m'-s)!} \left(\cos\frac{\theta}{2}\right)^{2J+m-m'-2s} \left(\sin\frac{\theta}{2}\right)^{m'-m+2s} \right], \tag{C.11}$$

with $d^J_{m'm}(\theta) = (-1)^{m-m'}\, d^J_{mm'}(\theta)$. Summing over λ_l for both the flip and the non-flip transitions and integrating over $\cos\theta$ we obtain the differential decay rate expression

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3}|V_{cb}|^2 \frac{|\overrightarrow{p}|}{m_B^2}\left(1-\frac{m_l^2}{q^2}\right)^2 q^2 \times \left\{ \left(1+\frac{2m_l^2}{q^2}\right)|H_{00}|^2 + \frac{3m_l^2}{4q^2}|H_{0t}|^2 + \left(2+\frac{m_l^2}{2q^2}\right)\left[|H_{0-}|^2 + |H_{0+}|^2\right] \right\}. \tag{C.12}$$

The non-vanishing hadronic helicity amplitudes (which survive the contractions $q^\mu\varepsilon_\mu(\lambda_W)$, $p_B^\mu\varepsilon_\mu(\lambda_W)$ and $p_D^\mu\varepsilon_\mu(\lambda_W)$) are [138]

$$H_{00} = \frac{1}{\sqrt{q^2}}F_V(q^2)2m_B|\overrightarrow{p}| \qquad \text{and} \qquad H_{0t} = \frac{1}{\sqrt{q^2}}F_S(q^2)\left(m_B^2 - m_D^2\right), \tag{C.13}$$

where F_V and F_S are the QCD form factors that parametrize the matrix element

$$\langle D(p_D)|\bar{c}\gamma^\mu b|B(p_B)\rangle = F_V(q^2)\left[p_B^\mu + p_D^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right] + F_S(q^2)\frac{m_B^2 - m_D^2}{q^2}q^\mu. \tag{C.14}$$

The remaining q^2-integration has to be done numerically due to the q^2-dependance of the form factors.

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ملخص

لغرض إعطاء تفسير لعدم انحفاظ عالمية نكهة اللبتون الملاحظة في اضمحلال الميزون B، أجريت دراسة تعتمد على النموذج 331 مع $\beta = 1/\sqrt{3}$ الذي يتطلب وجود خمس ثلاثيات ليبتون من أجل التمكن من إنشاء ثوابت اقتران والتي من شأنها أن تنتهك هذه الخاصية. في التحولات التي تتم إما عبر تيار محايد $b \longrightarrow sl^+l^-$ أو عبر تيار مشحون $b \longrightarrow cl\nu$، وجد أن النموذج المقترح لا يمكنه فقط شرح الإنحرافات عن النموذج المعياري (SM) الملاحظة تجريبياً، لكن أيضا، يمكن من خلاله أن تحدث إنتقالات تنتهك نكهة الليبتون. في الواقع، تم التوصل إلى ان النموذج قادر على استيعاب سيناريو NP (الفيزياء الحديثة) المفضل من خلال التحليل العام مع مساهمة كبيرة في نسبة معاملي ويلسون في المنطقة $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$ للتحولات التي تتم عبر تيار محايد، بشرط أن يسيطر على هذه الأخيرة تبادل البوزون المحايد الثقيل (الغريب) Z'_μ. أما في ما يخص التحولات التي تتم عبر تيارمشحون، ثبت توافق مساهمة NP في المؤثر $(\bar{c}\gamma_\mu b)\,(\bar{\nu}_l\gamma^\mu l)$ من خلال معامل $\mathcal{C}_{V_L}$ مع النتائج النظرية للتحليل العام بشرط أن يسيطر تبادل بوزون النموذج المعياري W_μ على الانتقال، وليس البوزون الثقيل، لكون الاقتران الناجم عن مزج هذا الاخير مع الفرميونات مهمل في مجال الطاقة المطلوبة.

الكلمات المفتاحية: الفيزياء الحديثة، ما وراء النموذج القياسي، عدم انحفاظ عالمية نكهة اللبتون، اضمحلال الميزون B.

Résumé

Afin de donner une explication à la violation de l'universalité de la saveur leptonique (LFUV) actuellement observée dans des désintégrations du méson B, une approche dépendante du modèle a été considérée. En effet, une étude a été menée dans ce contexte où une version non-minimale du modèle 331 avec $\beta = 1/\sqrt{3}$ a été choisie. Cette dernière exige la présence de cinq triplets de leptons afin de pouvoir générer des couplages qui violeraient l'univesalité leptonique. Dans les transitions à courant neutre $b \longrightarrow sl^+l^-$ ou à courant chargé $b \longrightarrow cl\nu$, il a été trouvé que, non seulement, le modèle pourrait expliquer les déviations du modèle standard (SM) observées expérimentalement, mais également, des transitions qui violeraient la saveur leptonique pourraient survenir. En effect, le modèle s'est avéré s'accommoder au scénario NP (nouvelle physique) favorisé par l'analyse globale avec une contribution signifiante dans le rapport des deux coefficients de Wilson dans la région $\mathcal{C}_9^\mu = -\mathcal{C}_{10}^\mu$ pour les transitions à courant neutre, pourvu que ces dernières soient dominées par l'échange du boson neutre lourd (exotique) Z'_μ, et celui léger du modèle standard Z_μ. Quant aux transitions à courant chargé, la contribution de NP à l'opérateur $(\bar{c}\gamma_\mu b)\,(\bar{\nu}_l\gamma^\mu l)$ avec un coefficient $\mathcal{C}_{V_L}$ s'est avérée en accord avec les résultats théoriques de l'analyse globale indépendante du modèle, pourvu que la transition soit dominée par l'échange du boson chargé léger du modèle standard W_μ et non celui lourd, vu que le couplage induit par le mixage du boson lourd avec les fermions est négligé à l'ordre d'énergie voulu.

Mots-Clés: Nouvelle Physique (NP), Théorie des Champs Effective (EFT), Extension du Modèle Standard, Violation de l'Universalité de la Saveur Leptonique (LFUV), Désintégration du méson B.